Homework 1: Link Layer

Due: 11:59 PM, Oct 2, 2017

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1 How long?

Assume the straight-line distance between NYC and Providence is 250Km, and the driving distance is 300Km. The speed of light in vacuum (and in the air) is $c = 3 \times 10^8 m/s$, and the speed of light in fiber is $2 \times 10^8 m/s$ (two thirds of c).

If we were to start an HFT (high frequency trading) company in Providence we would need to minimize the time it takes to receive and send information between NYC and Providence. Assume that we have 2 options: using a radio link which goes along the direct air path (it has a bandwidth of 10Mbps, and has 5 repeaters along the path (every repeater adds 0.1 ms to the latency of the line), or using a fiber cable which was laid along the road (it has a bandwidth of 1Gbps and no repeaters are necessary).

Assume that the path is symmetric, and you are using a stop-and-wait protocol in which the acknowledgments have negligible size. Also, don't worry about losses.

1. Which of the two links would you choose if all messages on the link are of size 100B?

- Latency on link 1: The time to send a 100B message and receive the acknowledgment will be the time to transmit 100B plus the round-trip time, as per the diagram.
  - t = Time to transmit 100B at 10Mbps $= 10^{-7}s/b \times 8b/B \times 10^2 B = 8 \times 10^{-5}s$.
  - l = Round-trip time: $(\frac{2.5\times10^5 m}{3\times10^8 m/s} + 5 \times 10^{-4}s) \times 2 =$
    $(8.33 \times 10^{-4}s + 5 \times 10^{-4}s) \times 2 =$
    $2.667 \times 10^{-3}s$
  - T = Total time $= 2.747 \times 10^{-3}s$.
- Latency on link 2:
  - t = Time to transmit 100B at 1Gbps $= 10^{-9}s/b \times 8b/B \times 10^2 B = 8 \times 10^{-7}s$.
  - l = Round-trip time: $(3 \times 10^5 m/2 \times 10^8 m/s) \times 2 =$
    $(1.5 \times 10^{-3}s) \times 2 =$
    $3 \times 10^{-3}s$
  - T = Total time $= 3.0008 \times 10^{-3}s$.

We would prefer the first link.

2. How many such messages would you need to send in a sliding window protocol to fully utilize the link? Draw a time diagram similar to the ones we saw in class to illustrate your calculations.

See the diagram for the previous item. T is calculated from the previous item, and so is t. The window size is T/t, or how many messages can you send before the acknowledgment for the first message arrives.

For the first link, $T/t = 2.747 \times 10^{-3}s/8 \times 10^{-5} = 34.3$ messages.

For the second link, $T/t = 3.0008 \times 10^{-3}/8 \times 10^{-7} = 3751$ messages.

3. Which of the two links would you choose if all messages on the link are of size 1000B? If the message size is 1000B, then t changes, but not l.

For the first link, $t = 10^{-7}s/b \times 8b/B \times 10^3B = 8 \times 10^{-4}s$.
$T = 2.667 \times 10^{-3} + 8 \times 10^{-4} = 3.467 \times 10^{-3}s$.

For the second link, $t = 10^{-9}s/b \times 8b/B \times 10^3B = 8 \times 10^{-6}s$.
$T = 3 \times 10^{-3} + 8 \times 10^{-6} = 3.008 \times 10^{-3}s$.

In this case, we would prefer the second link.

Explain and show the calculations for your answers.

2 Modulation

Suppose you are designing a scheme for transmission of a wireless signal, and you need to send data at 1Gbps (10^9 bits/s). The bandwidth of the channel is 80MHz, and you measure the noise floor to be -90dBm (dBm is a way to express power as a logarithmic ratio to a reference power of 1mW: p dBm $= 10\log_{10}\frac{PmW}{1mW}$. Thus, $P(\text{mW}) = 10^{\frac{\text{pdBm}}{10}}$, and -90dBm = $10^{-9}mW$).

1. Given the channel bandwidth, what is the minimum number of levels (M) that you need to be able to achieve the desired rate? (M should be an integer power of 2).

Hartley's law states that we need $2B\log_2(M) \geq C$ (ie, this should be at least the desired capacity).
$\log_2(M) \geq C/2B$
$\log_2(M) \geq 10^9/160 \times 10^6 = 6.25$
The smallest power of two for which this is true is 7, and M = 128.

2. What is the minimum signal strength at the receiver, in mW, to enable this number of levels in your transmission?

Now we need to look at Shannon's law as well.
$M \leq \sqrt{1 + S/N}$
$1 + S/N \geq M^2$
$S/N \geq M^2 - 1$
$S/N \geq 2^{14} - 1 = 16383$

The noise floor (N) is $10^{-9} mW$, which means that the signal has to be at least $10^{-9} \times 1.6383 \times 10^4 = 1.6383 \times 10^{-5} mW$.

3 Error correction

Parity bits

In the slides we describe a 2-D parity code that works like this: for each group of 7 bits, add a parity bit. Then, add a parity byte after the last byte, to check the parity of the columns.

```
0 0 1 1 0 1 0  1
1 1 0 1 1 1 0  1
0 1 1 0 1 1 0  0
1 1 1 0 0 1 1  1
0 0 0 1 1 0 1  1
1 1 1 1 1 1 0  0

1 0 0 0 0 1 0  0
```

1. Starting from a valid encoding (one in which all the parity bits are correct), what is the smallest number of bits you can flip to go to another valid encoding? (Remember you can also flip parity bits, as in, errors could also corrupt the parity bits).

4. The smallest number of bits you can flip to reach another valid encoding is any 4 bits that form the corner of a rectangle.

2. Use this number to justify the error correction / detection abilities of this code.

This code can dectect errors of up to 3 bits, and correct errors of up to 1 bit. (Errors of two bits will get detected, but it will be ambiguous which valid configuration is closest.

3. Can you give an example of a 4 bit error that this code cannot detect, as well as an example of a 4 bit error that this code can detect?

This code cannot detect 4 bits that flip aligned with the corner of a rectangle.

The code can detect 4-bit errors such that the bits do not form a rectangle.

4. What is the rate of this particular code (the number of useful bits divided by the total number of bits)? This code will work for any matrix size starting at 2x2. This example is a 7x6 matrix. If you fix that you want lines of 8 bits (7+1), what factors would you take into account in selecting the number of rows in the matrix? (Assume you have a long message, and that you will then add the extra line every r bytes). Hint: you have to assume something about the environment.

This code adds 14 bits to every message. The rate is $42/(42 + 14) = 0.75$.

Changing the size of the message will change the rate of the code. If the error probability is low in the channel, we can make the messages bigger, and decrease the rate.

On the other hand, if the channel is dropping many packets, or parts of packets, then we should decrease the number of rows covered by the last row.

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`https://piazza.com/brown/fall2017/cs168`.