Introduction to Computer Vision

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Motion estimation

Goals

- Introduce motion estimation
  - Principles, assumptions and foundations

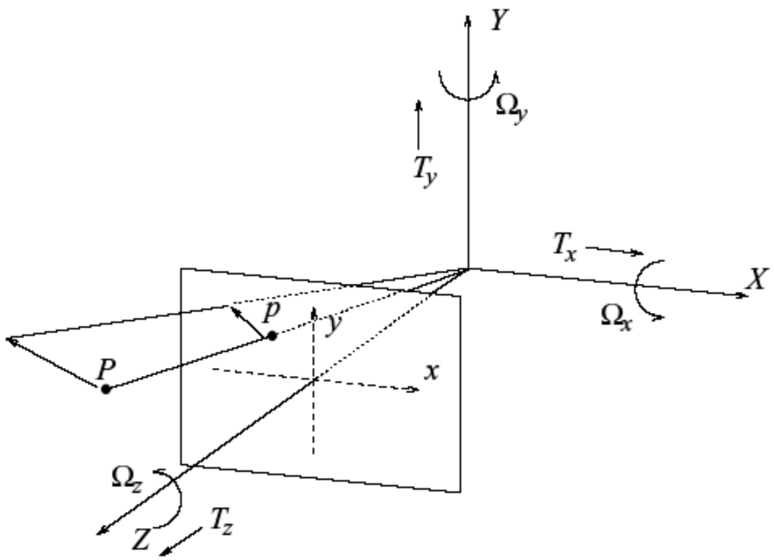
Readings

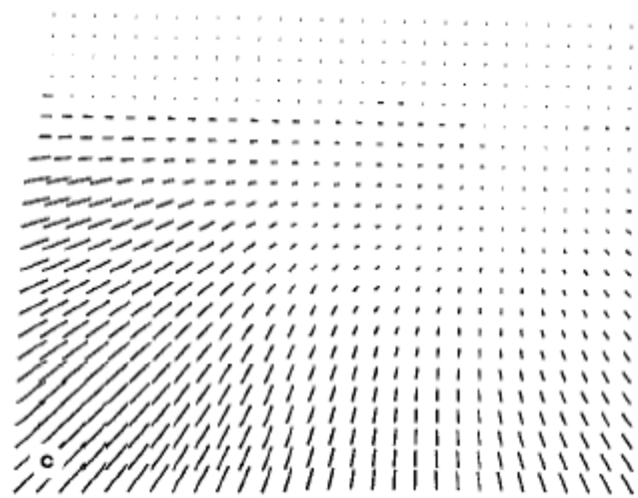
- Szeliksi: 8.1, 8.2, 8.4

Optical Flow Field

Image irradiance at time *t* and location **x**=(*x*, *y*)

$$I(x, y, t)$$

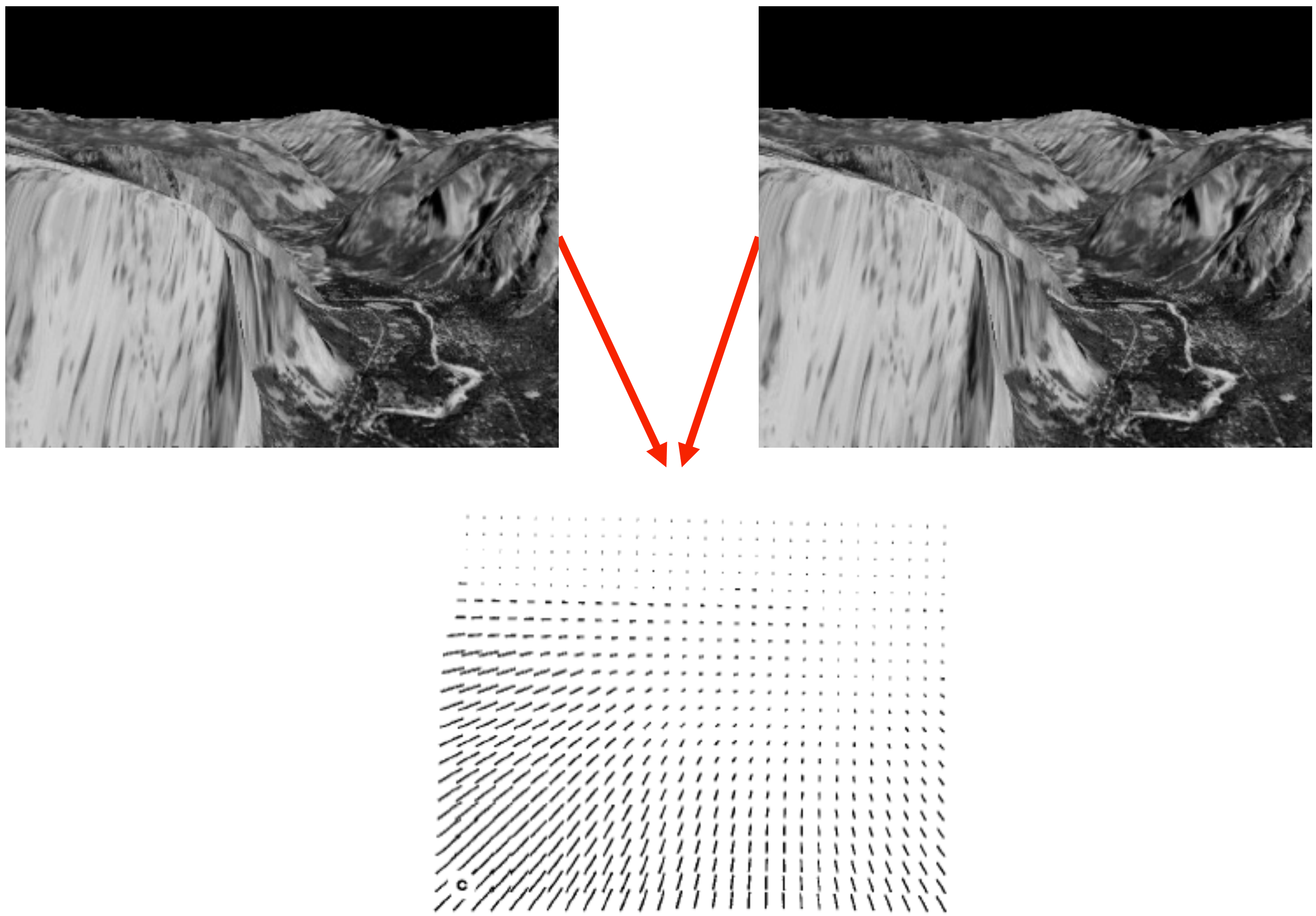




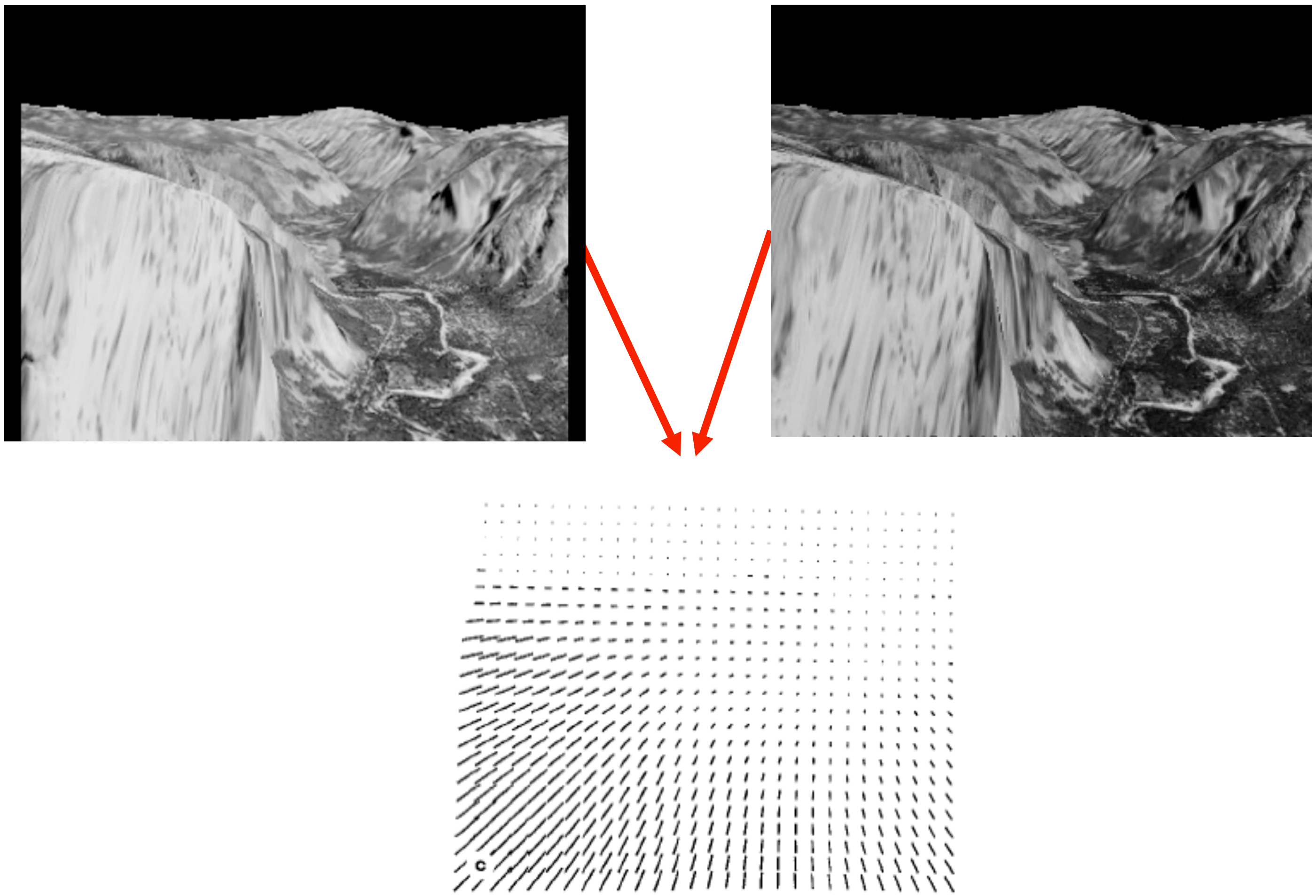
$u(x, y)$ Horizontal component

$v(x, y)$ Vertical component

Problem



Problem



Thought Experiment 1

Lambertian (matte) ball rotating in 3D

What does the 2D motion field look like?

What does the 2D optical flow field look like?

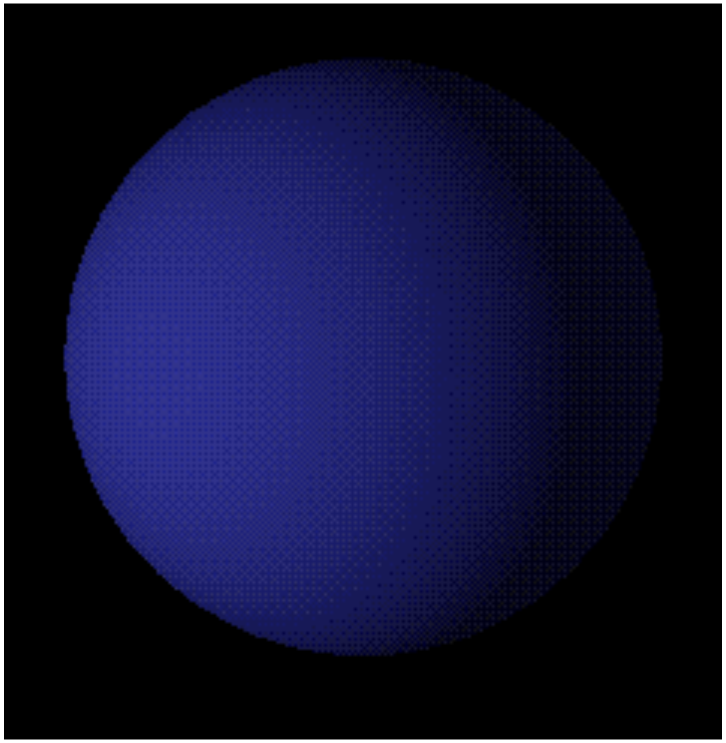


Image source: http://www.evl.uic.edu/aej/488/lecture12.html

Thought Experiment 2

Stationary Lambertian (matte) ball, moving light source.

What does the 2D motion field look like?

What does the 2D optical flow field look like?

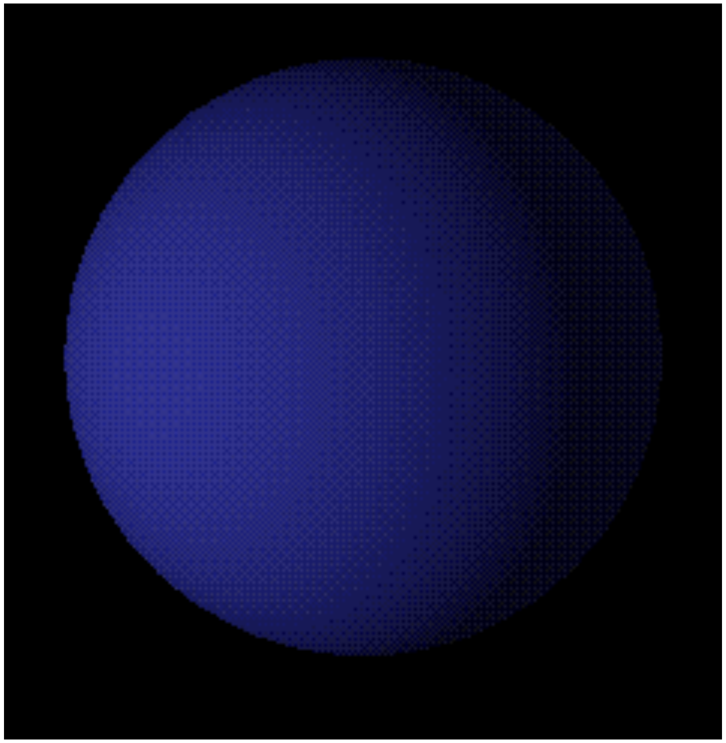
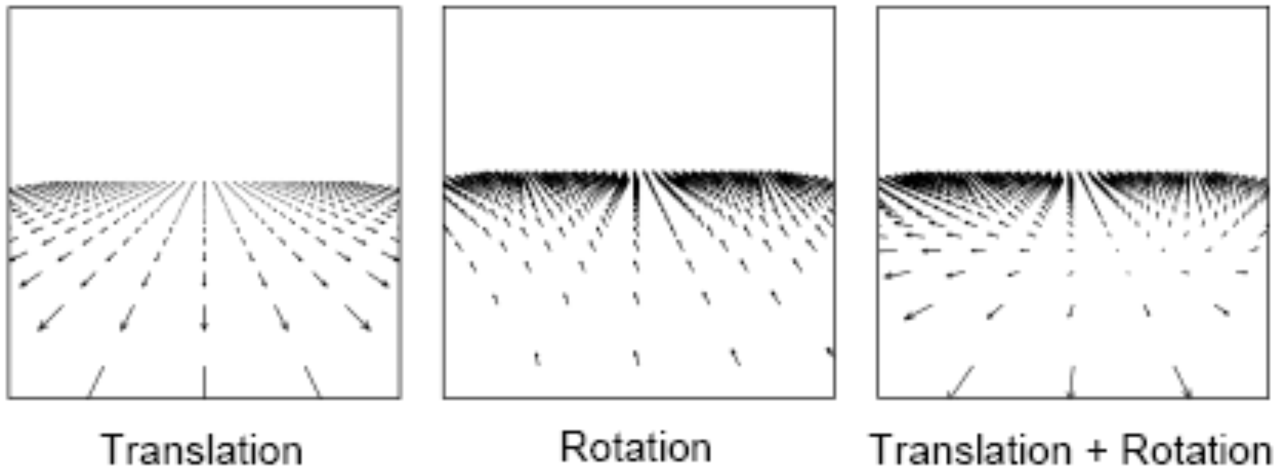


Image source: http://www.evl.uic.edu/aej/488/lecture12.html

Why Compute Flow?



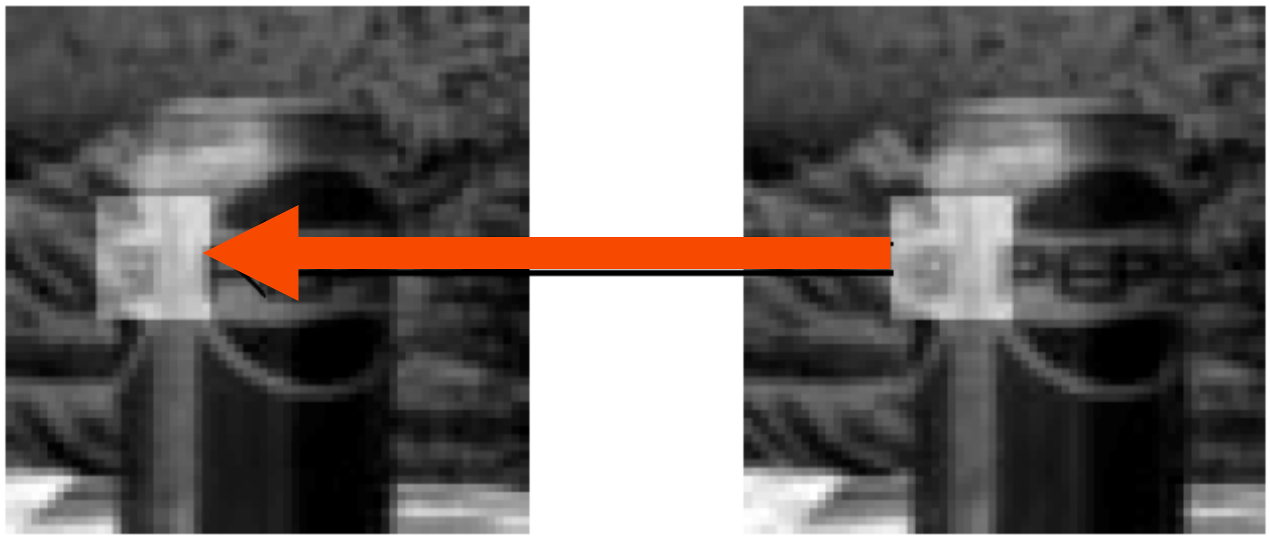
Tells us something (maybe ambiguous) about the 3D structure of the world and the motion (if any) of the observer.

We'll see many applications!

Computing optical flow

- What properties of the world can we use?
  - What cues?
  - What assumptions?
  - What constraints?
  - How do we formalize this?

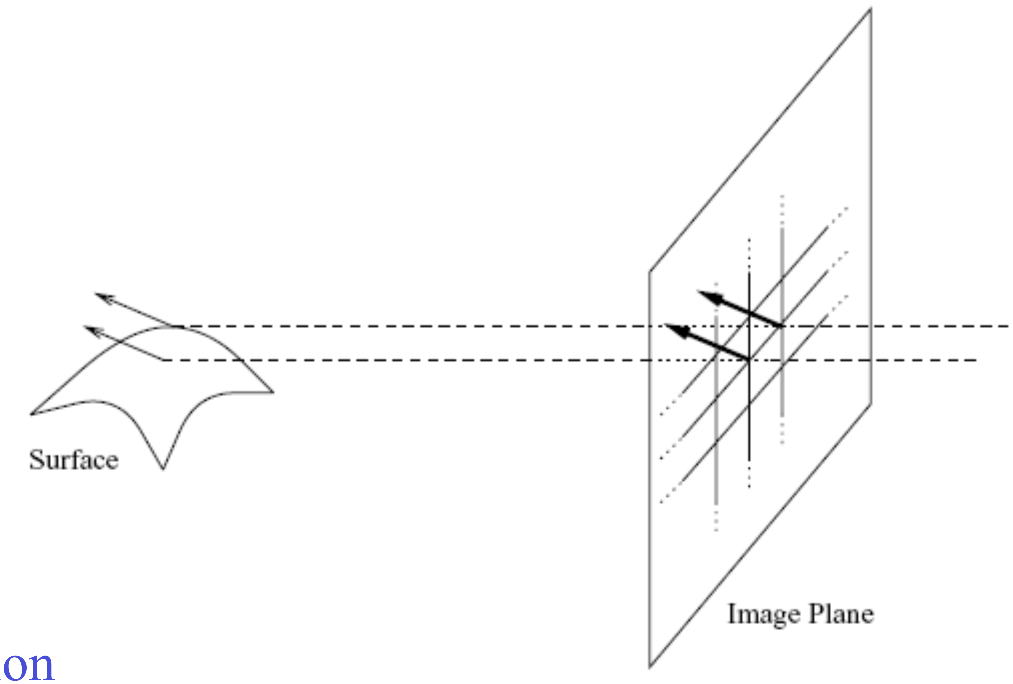
Brightness Constancy



$$I(x+u, y+v, t+1) = I(x, y, t)$$

(assumption)

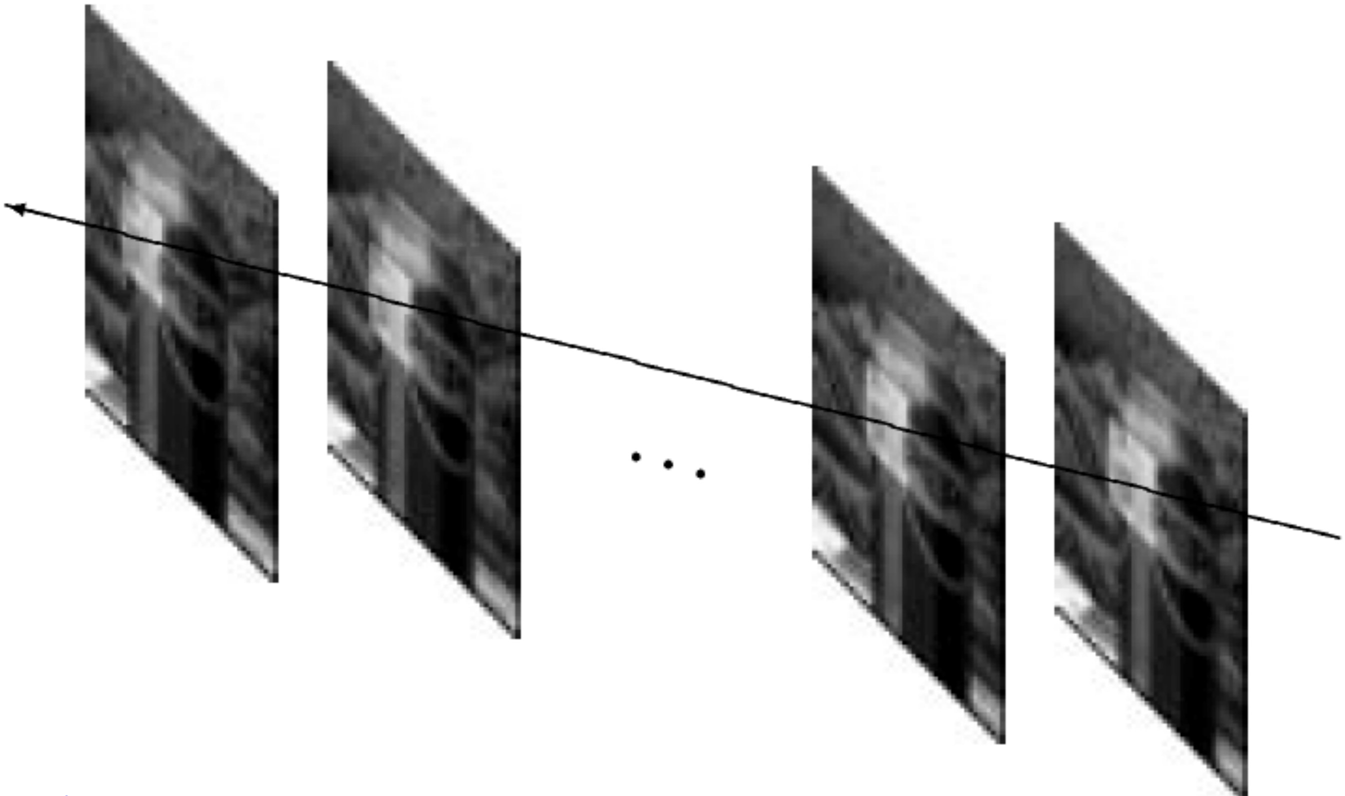
Spatial Coherence



Assumption

* Neighboring points in the scene typically belong to the same surface and hence typically have similar motions.
* Since they also project to nearby points in the image, we expect spatial coherence in image flow.

Temporal Persistence

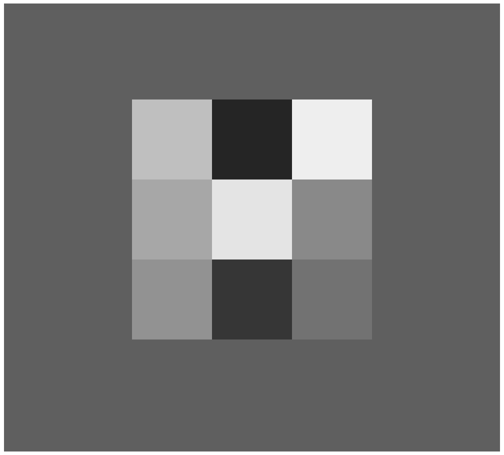


Assumption:

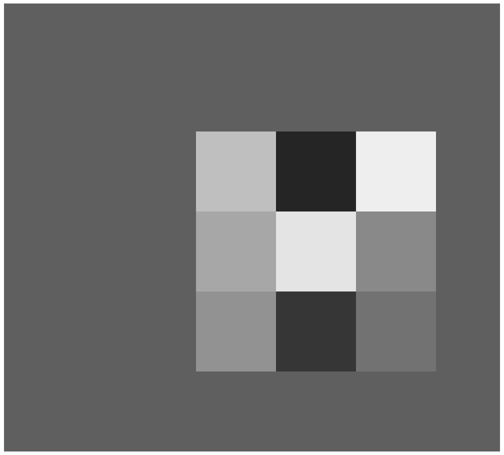
The image motion of a surface patch changes gradually over time.

Minimize Brightness Difference

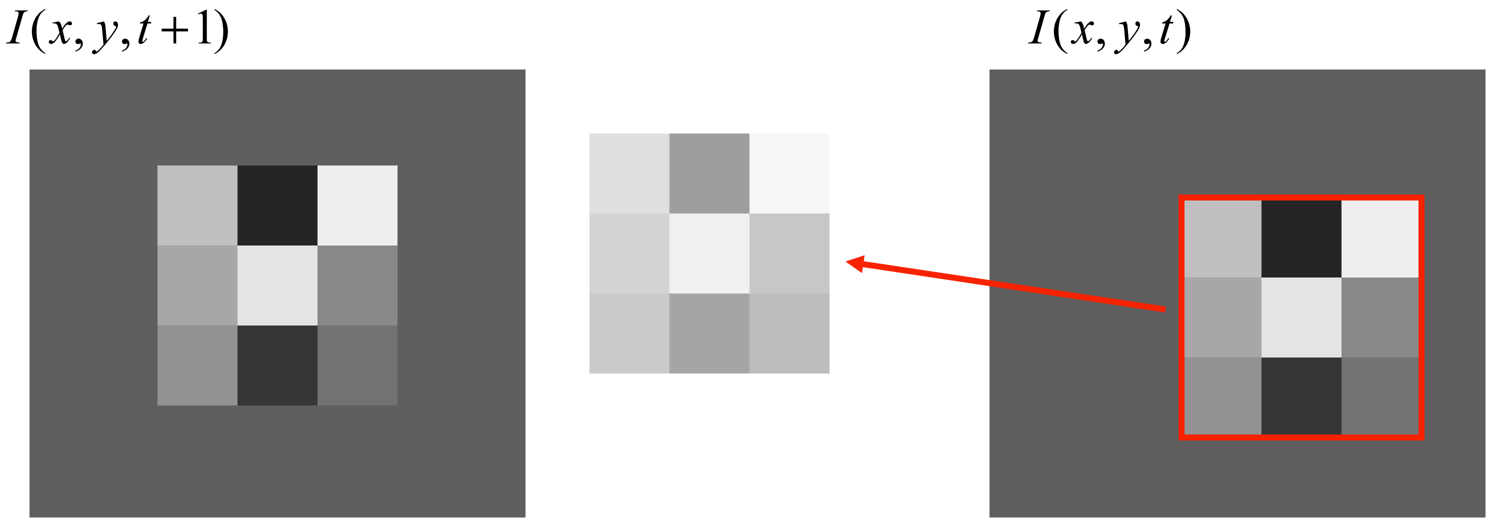
$I(x, y, t+1)$



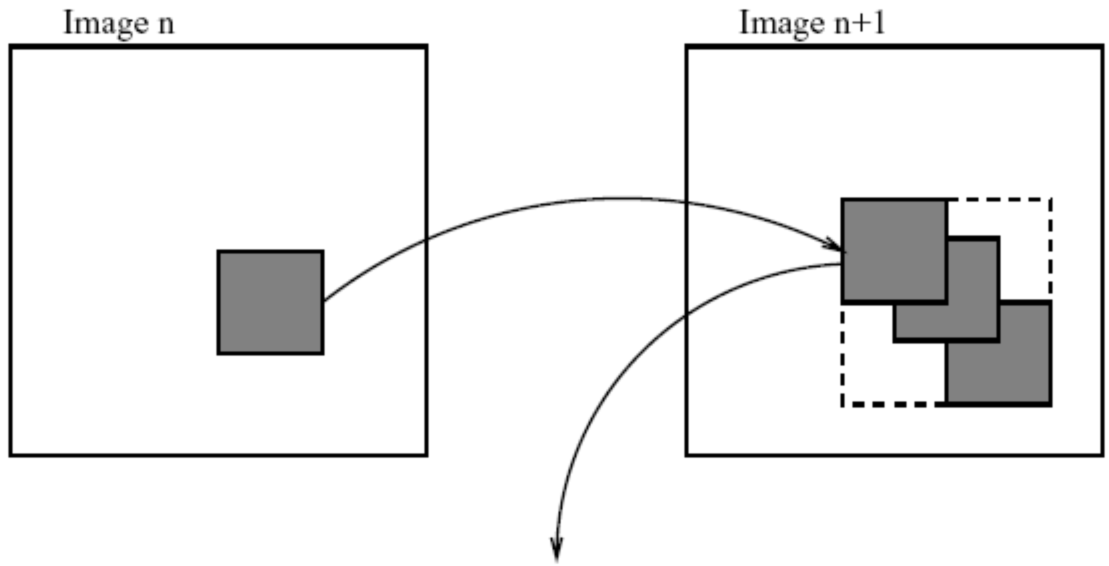
$I(x, y, t)$



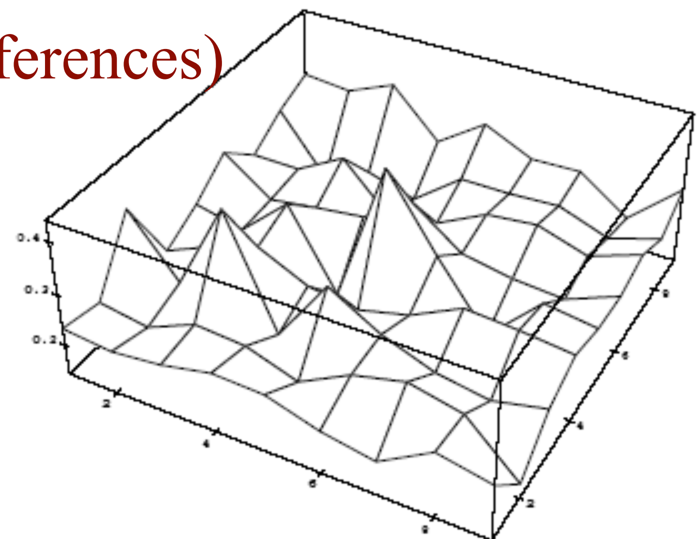
Minimize Brightness Difference



$$E_{SSD}(u,v) = \sum_{x,y \in R} \left(I(x+u, y+v, t+1) - I(x,y,t)\right)^2$$



SSD surface
(sum of squared differences)



$$E_{SSD}(u,v) = \sum_{x,y \in R} \left(I(x+u, y+v, t+1) - I(x,y,t)\right)^2$$

How can we optimize over u,v?

$$E_{SSD}(u,v) = \sum_{x,y \in R} (I(x+u, y+v, t+1) - I(x,y,t))^2$$

How do we find the minimum of $E_{SSD}(u,v)$?

What is the problem with the equation in this form?

Can we approximate this?

$$E_{SSD}(u,v) = \sum_{x,y \in R} (I(x+u, y+v, t+1) - I(x,y,t))^2$$

$$I(x+udt, y+vdt, t+dt)$$

$$I(x+dx, y+dy, t+dt)$$

Taylor Series Approximation

$$I(x,y,t) + dx\frac{\partial}{\partial x}I(x,y,t) + dy\frac{\partial}{\partial y}I(x,y,t) + dt\frac{\partial}{\partial t}I(x,y,t) + \varepsilon$$

Taylor Series Approximation

$$I(x+u, y+v, t+1)$$

$$dx = u,\ dy = v,\ dt = 1$$

Assume *u, v, dt small*

Assume brightness varies *smoothly* with *x, y, t*

$$I(x,y,t) + dx\frac{\partial}{\partial x}I(x,y,t) + dy\frac{\partial}{\partial y}I(x,y,t) + dt\frac{\partial}{\partial t}I(x,y,t) + \varepsilon$$

Brightness Constancy

$$E_{SSD}(u,v) = \sum_{x,y \in R} (I(x+u, y+v, t+1) - I(x,y,t))^2$$

$$\cancel{I(x,y,t)} + dx\frac{\partial}{\partial x}I(x,y,t) + dy\frac{\partial}{\partial y}I(x,y,t) + dt\frac{\partial}{\partial t}I(x,y,t) - \cancel{I(x,y,t)} = 0$$

Divide through by *dt*

$$u\frac{\partial}{\partial x}I(x,y,t) + v\frac{\partial}{\partial y}I(x,y,t) + \frac{\partial}{\partial t}I(x,y,t) = 0$$

Approximating SSD

$$E_{SSD}(u,v) = \sum_{x,y \in R} (I(x+u, y+v, t+1) - I(x,y,t))^2$$

$$I_x(x,y,t)u + I_y(x,y,t)v + I_t(x,y,t)$$

"Optical flow constraint equation"

$$I_x u + I_y v + I_t = 0$$

Notation

$$I_x u + I_y v + I_t = 0$$

↓

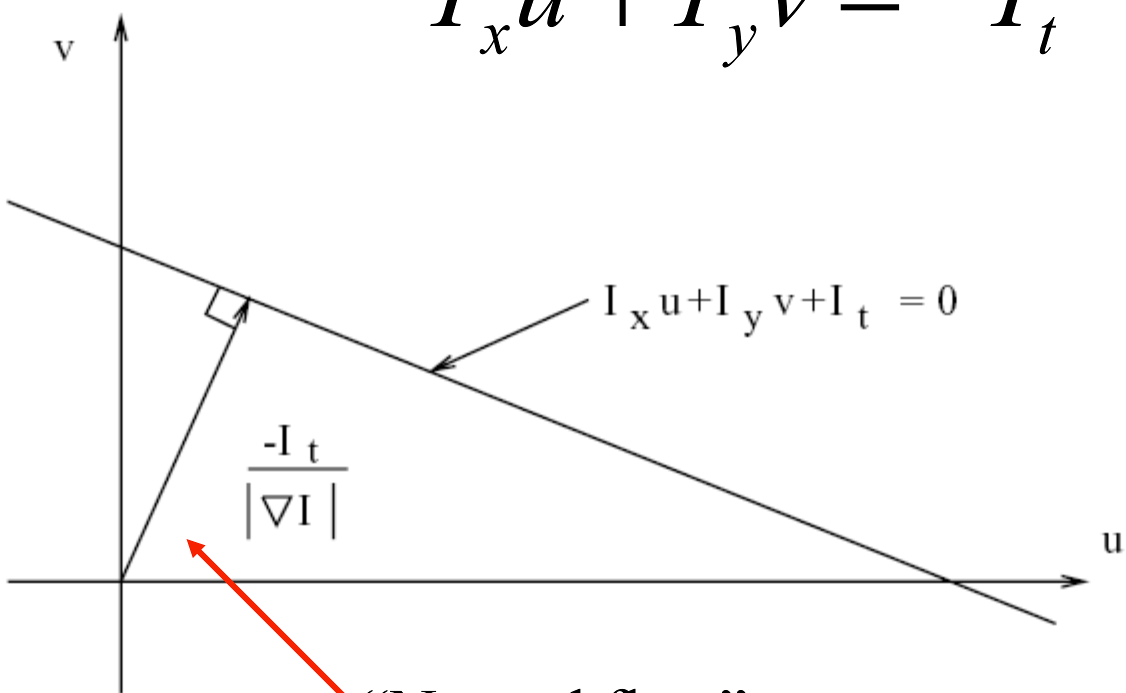
$$\nabla I^T \mathbf{u} = -I_t$$

$$\nabla I = \begin{bmatrix} I_x \\ I_y \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} u \\ v \end{bmatrix}$$

OFCE

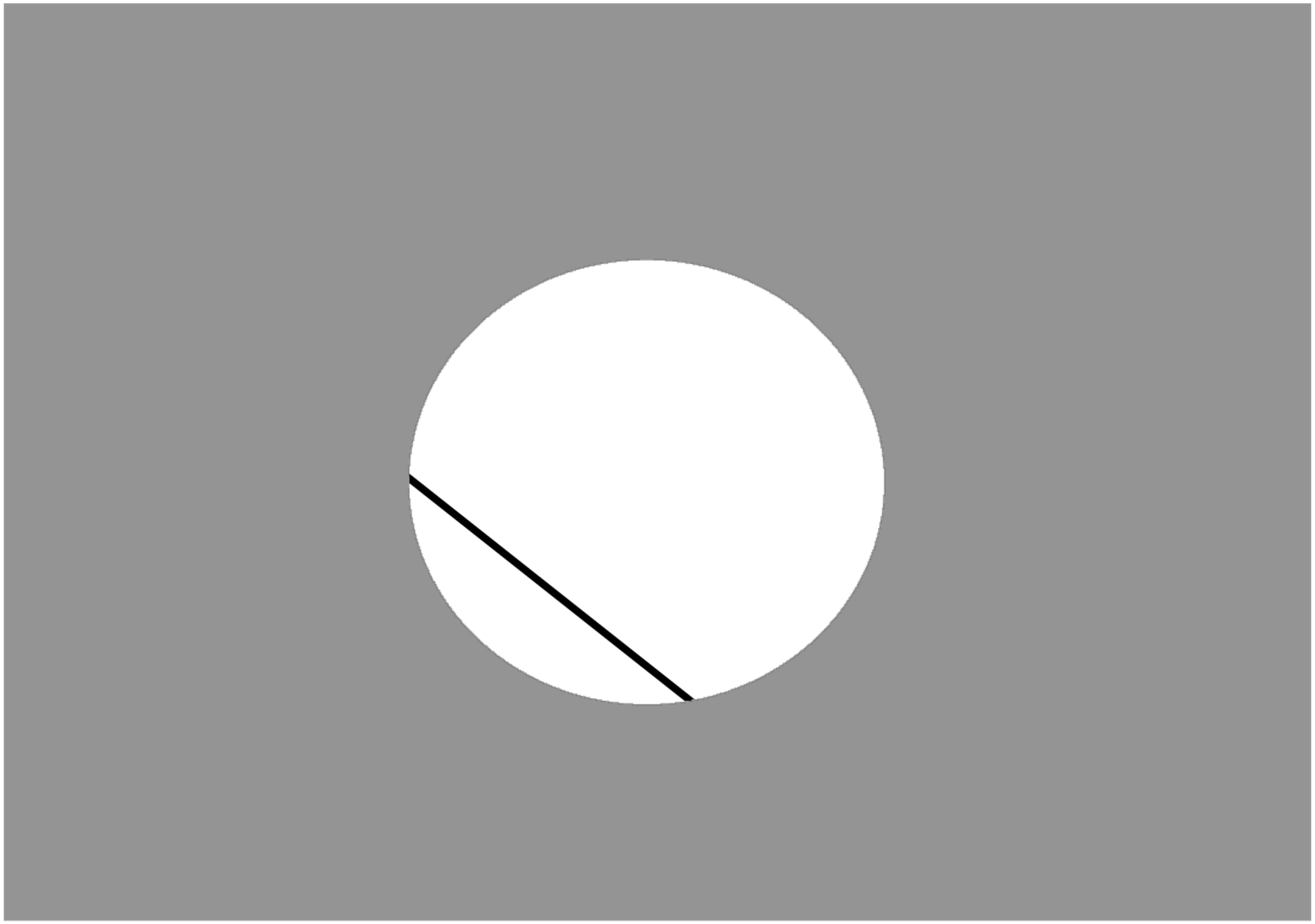
At a single image pixel, we get a line:

$$I_x u + I_y v = -I_t$$



"Normal flow"

Aperture Problem



Aperture Problem