# Introduction to Computer Vision 

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Stereo

## Goals

- Today
- Binocular stereo
- Friday
- Either object recognition or human shape and pose.







## Binocular Stereo <br> $\searrow$



## Binocular Stereo



## Binocular Stereo



## Binocular Stereo



## Binocular Stereo



## Binocular Stereo



## Binocular Stereo

Left


binocular disparity

From known geometry of the cameras and estimated disparity, recover depth in the scene


## Stereo Geometry



Scharstein
camera camera

## Stereo Geometry



## Stereo Geometry



## Stereo Geometry

## Disparity d <br> = difference in image position



## Stereo Geometry



## Stereo Geometry



## Binocular Disparity

$\mathrm{Z}(\mathrm{x}, \mathrm{y})$ is depth at pixel $(\mathrm{x}, \mathrm{y})$ $\mathrm{d}(\mathrm{x}, \mathrm{y})$ is disparity

Estimate:

$$
Z(x, y)=\frac{f b}{d(x, y)}
$$

Left


## Binocular Disparity

$Z(x, y)$ is depth at pixel ( $x, y$ ) $\mathrm{d}(\mathrm{x}, \mathrm{y})$ is disparity

Estimate:

$$
Z(x, y)=\frac{f b}{d(x, y)}
$$

Left


Do I need to consider this region?


## Epipolar Geometry



## Epipolar Geometry



## Epipolar Geometry



## Epipolar Geometry



## Epipolar Geometry

Possible matches for $\mathrm{p}_{1}$ are constrained to lie along the epipolar line in the other image


## Epipole

- Every plane through the baseline is an epipolar plane, and determines a pair of epipolar lines in the two images
- Two systems of epipolar lines are obtained, each system intersects in a point, the epipole
- The epipole is the projection of the center of the other camera


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## Rectification



Rectification aligns epipolar lines with scanlines.

- warp images

Szeliski and Fleet

## Rectification



Szeliski and Fleet
CS143 Intro to Computer Vision
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## Matching



* Matching only has to occur along epipolar lines.
* Now in the simpler binocular case where the cameras are pointing forward.
* Compare with optical flow.


## Stereo Correspondence

- Search over disparity to find correspondences
- Range of disparities to search over can change dramatically within a single image pair.



## Correspondence Using SSD



# Sum of Squared (Pixel) Differences 


$w_{L}$ and $w_{R}$ are corresponding $m$ by $m$ windows of pixels.

The SSD cost measures the intensity differenceas a function of disparity :
$\operatorname{SSD}_{r}(x, y, d)=\sum_{\left(x^{\prime}, y^{\prime}\right) \in W_{m}(x, y)}\left(I_{L}\left(x^{\prime}, y^{\prime}\right)-I_{R}\left(x^{\prime}-d, y^{\prime}\right)\right\}^{2}$

## Dealing with ambiguity



Many repeated structures


Baseline b 2b 3b 4b 5b 6b 7b 8b 9b

* Collect multiple views with different baselines.
M. Okutomi, T. Kanade, Multiple-Baseline Stereo

(f)

(g)


Fig. 5. SSD values versus inverse distance: (a) $B=b$; (b) $B=2 b$; (c) $B=3 b$; (d) $B=4 b$; (e) $B=5 b$; (f) $B=6 b$; (g) $B=7 b$; (h) $B=8 b$. $B=3 b$; (d) $B=4 b$; (e) $B=5 b$; (f) $B=6 b$; (g) $B=7 b$; (h) $B=8 b$.
The horizontal axis is normalized such that $8 b F=1$.


Fig. 6. Combining two stereo pairs with different baselines.


Fig. 7. Combining multiple baseline stereo pairs.
Kanade, Multiple-Baseline Stereo

## Matching

- Even when the cameras are identical models, there can be differences in gain and sensitivity.
- The cameras do not see exactly the same surfaces, so their overall light levels can differ.
- occlusion

$$
E_{r}(x, y, d)=\sum_{\left(x^{\prime}, y^{\prime}\right) \in W_{m}(x, y)} \rho\left(I_{L}\left(x^{\prime}, y^{\prime}\right)-I_{R}\left(x^{\prime}-d, y^{\prime}\right)\right)
$$

Robust matching function.
Looks like optical flow. Why don't we linearize this?

## Matching

- Even when the cameras are identical models, there can be differences in gain and sensitivity.
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- occlusion

$$
E_{r}(x, y, d, a, b)=\sum_{\left(x^{\prime}, y^{\prime}\right) \in W_{m}(x, y)} \rho\left(I_{L}\left(x^{\prime}, y^{\prime}\right)-\left(a I_{R}\left(x^{\prime}-d, y^{\prime}\right)+b\right)\right)
$$

Can add parameters to model illumination differences between cameras.

## Correspondence Using SSD



Disparity Map


Images courtesy of Point Grey Research

## Bayesian Interpretation



$$
\mathrm{p}_{\mathrm{M}}\left(\mathbf{d} \mid \mathrm{I}_{\mathrm{L}}, \mathrm{I}_{\mathrm{R}}\right)
$$

How do we proceed?

## Bayesian inference

Prior model

$$
p_{\mathrm{P}}(\boldsymbol{d})
$$

Likelihood model $\quad p_{\mathrm{M}}\left(\mathrm{I}_{\mathrm{L}}, \mathrm{I}_{\mathrm{R}} \mid \boldsymbol{d}\right)$
Posterior model

$$
p\left(\boldsymbol{d} \mid \mathrm{I}_{\mathrm{L}}, \mathrm{I}_{\mathrm{R}}\right)=k p_{\mathrm{M}}\left(\mathrm{I}_{\mathrm{L}}, \mathrm{I}_{\mathrm{R}} \mid \boldsymbol{d}\right) p_{\mathrm{P}}(\boldsymbol{d})
$$

Maximum a Posteriori (MAP estimate): $\operatorname{maximize} p\left(\boldsymbol{d} \mid \mathrm{I}_{\mathrm{L}}, \mathrm{I}_{\mathrm{R}}\right)$

