Straight Lines and Hough

Computer Vision

CS 143, Brown

James Hays

Many slides from Derek Hoiem, Lana Lazebnik, Steve Seitz, David Forsyth, David Lowe, Fei-Fei Li

Project 1

- A few project highlights
- Common mistakes
  - Gaussian pyramid stores blurred images.
  - Laplacian pyramid doesn’t have all the information needed for correct reconstruction.
  - Absolute paths in source code or html
  - Many of the results not very convincing because high and low frequencies are too different

Project 2

- Questions?

Canny edge detector

1. Filter image with x, y derivatives of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   – Thin multi-pixel wide “ridges” down to single pixel width
4. Thresholding and linking (hysteresis):
   – Define two thresholds: low and high
   – Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB: edge(image, ‘canny’)

Source: D. Lowe, L. Fei-Fei

Finding straight lines

- One solution: try many possible lines and see how many points each line passes through

- Hough transform provides a fast way to do this

Hough transform

- An early type of voting scheme
- General outline:
  - Discretize parameter space into bins
  - For each feature point in the image, put a vote in every bin in the parameter space that could have generated this point
  - Find bins that have the most votes

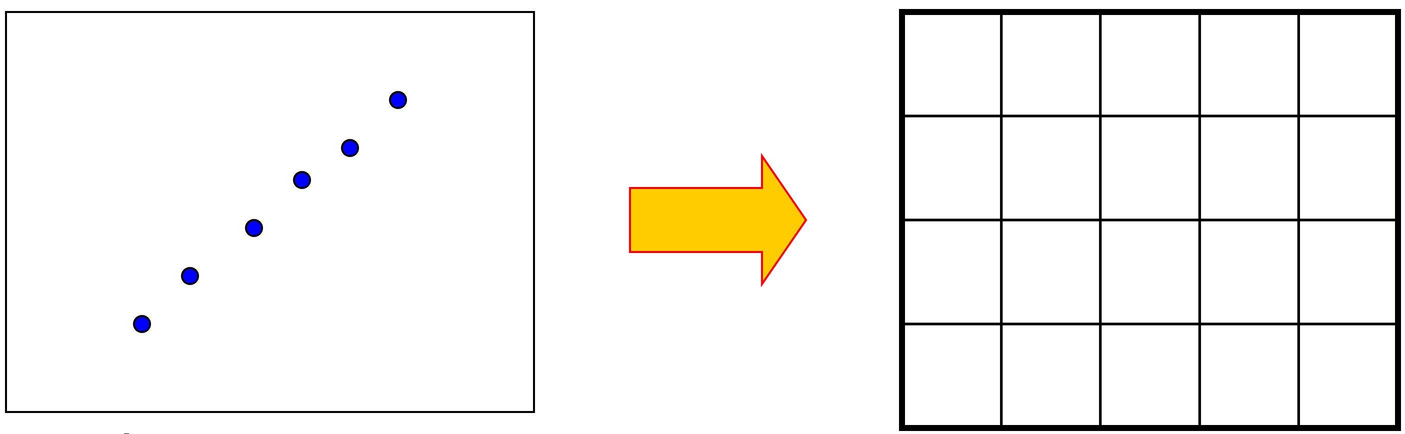


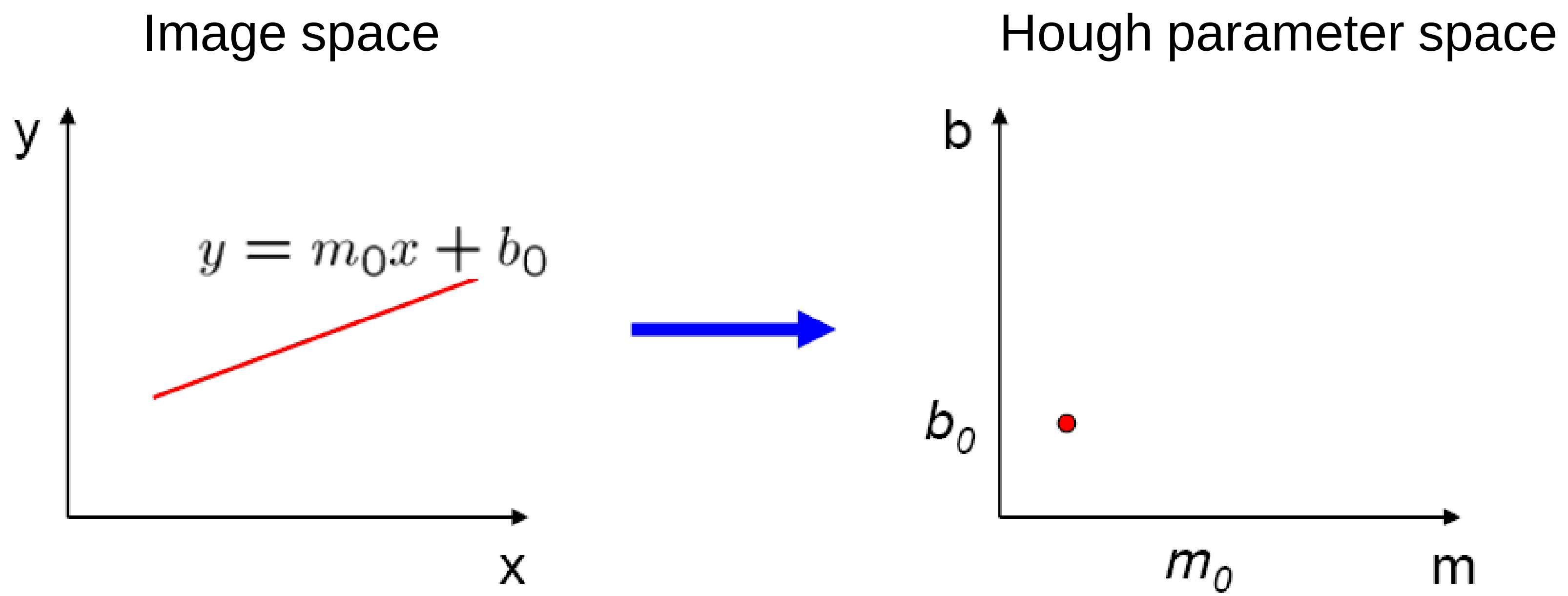
Image space

Hough parameter space

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Parameter space representation

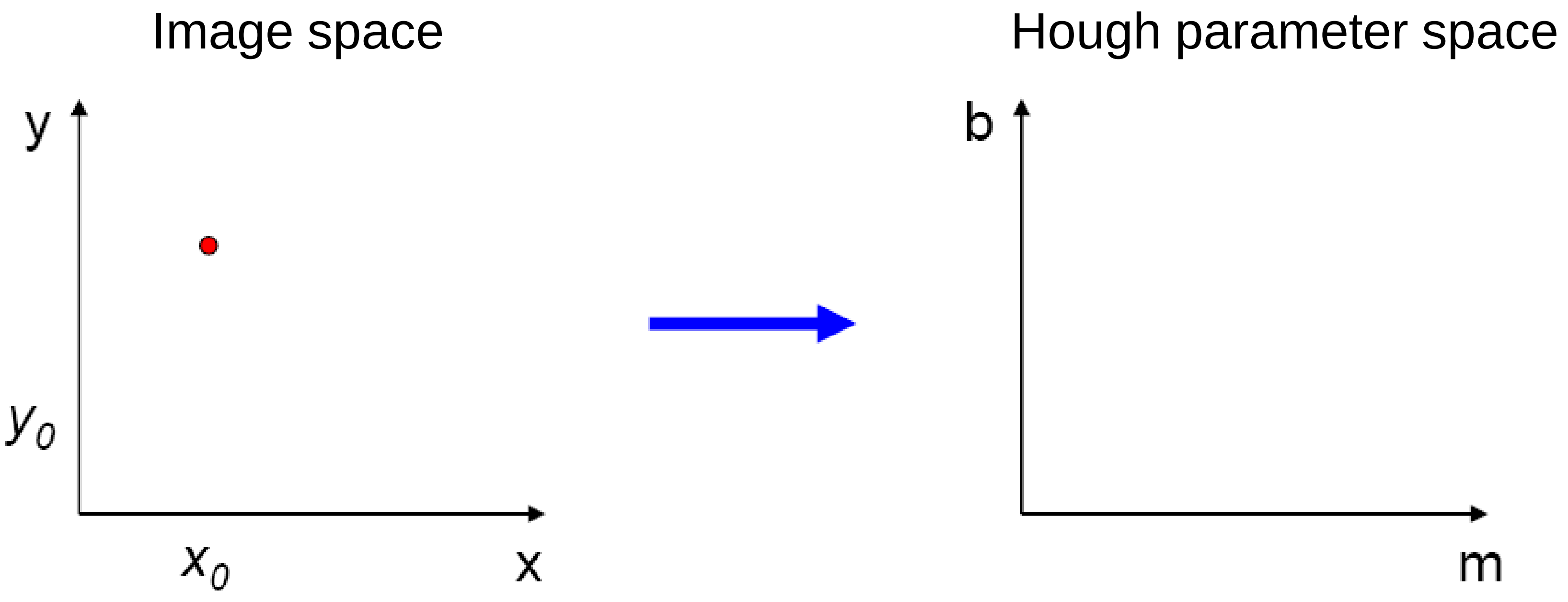
- A line in the image corresponds to a point in Hough space



Source: S. Seitz

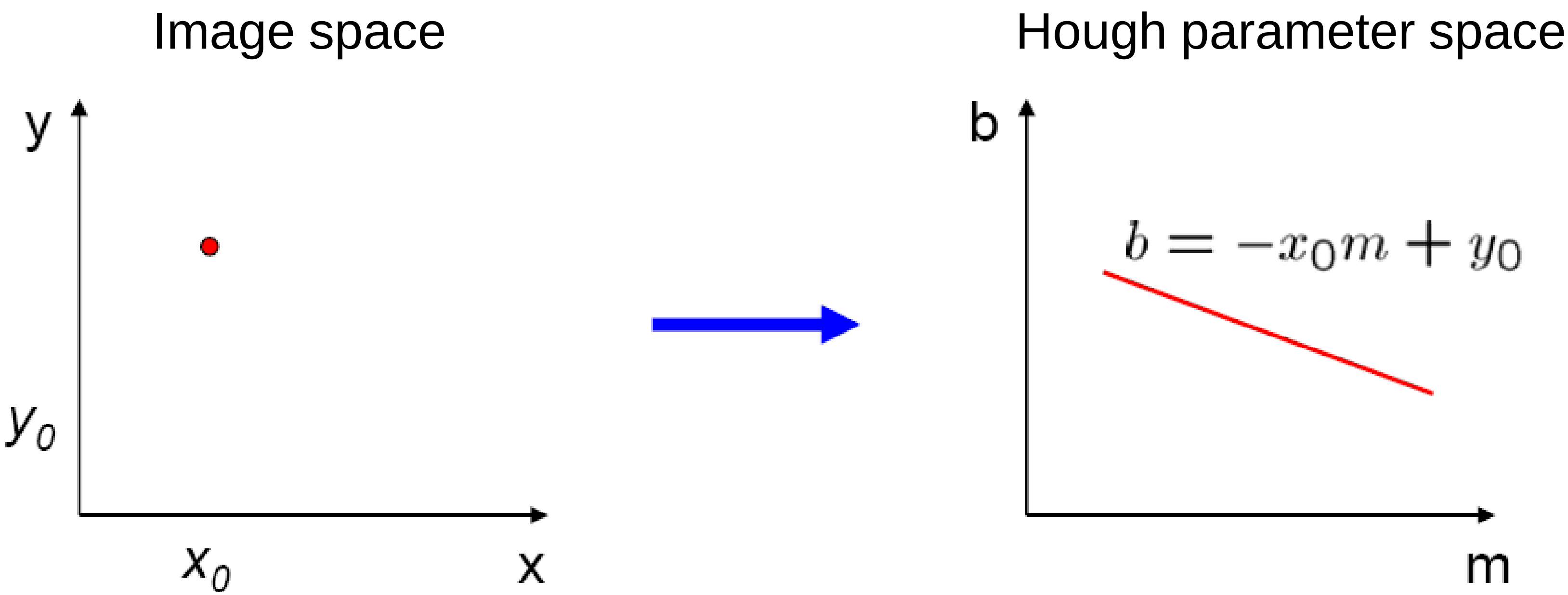
Parameter space representation

- What does a point (x_0, y_0) in the image space map to in the Hough space?



Parameter space representation

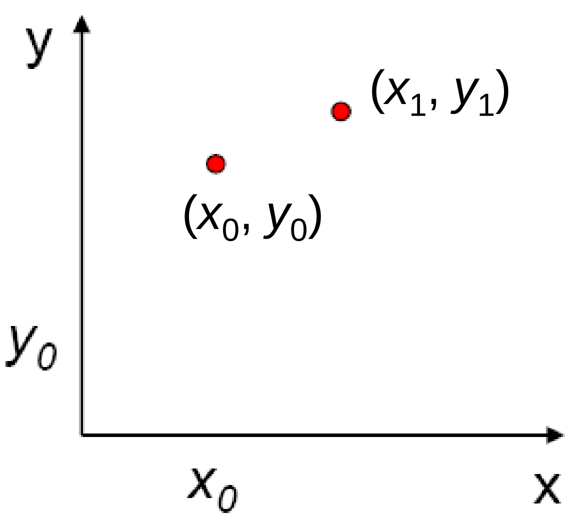
- What does a point (x_0, y_0) in the image space map to in the Hough space?
  - Answer: the solutions of $b = -x_0 m + y_0$
  - This is a line in Hough space



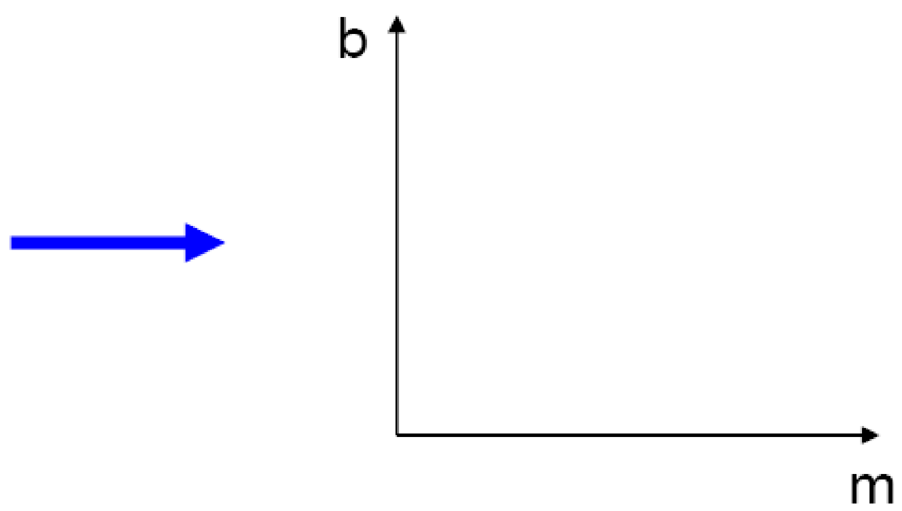
Parameter space representation

- Where is the line that contains both (x_0, y_0) and (x_1, y_1)?

Image space

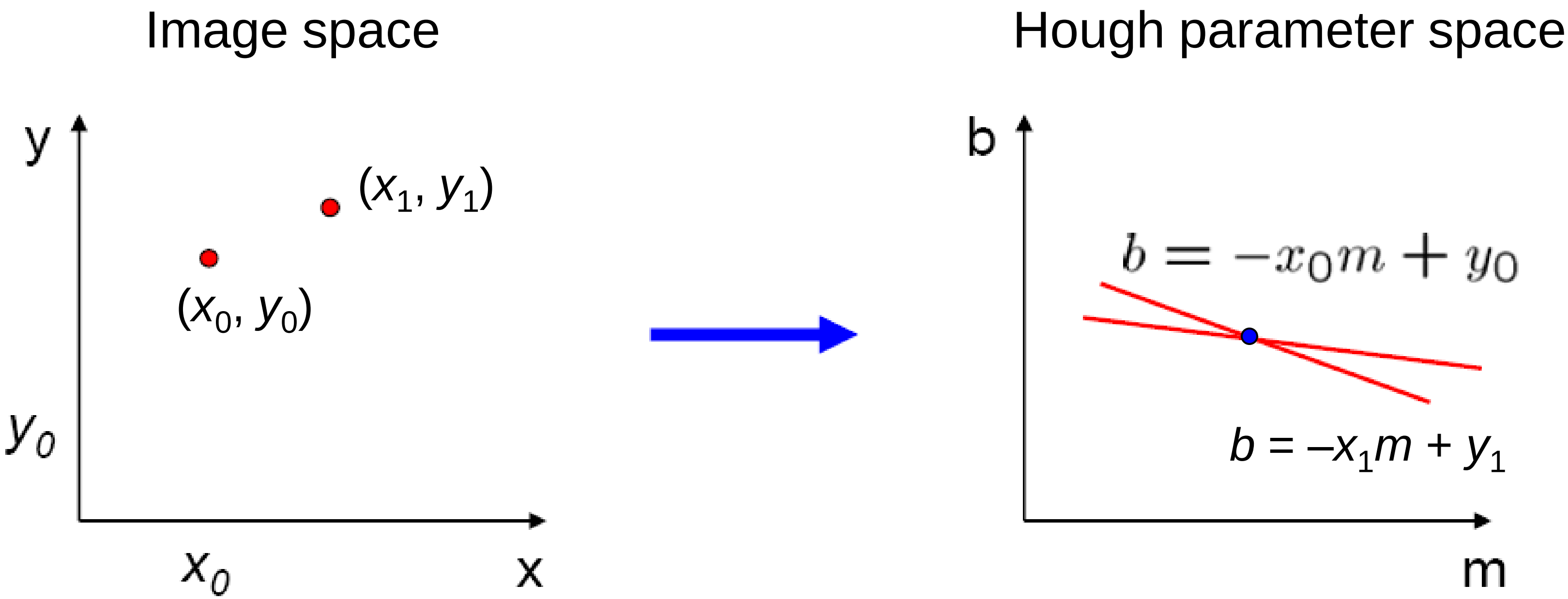


Hough parameter space



Parameter space representation

- Where is the line that contains both (x_0, y_0) and (x_1, y_1)?
  - It is the intersection of the lines $b = -x_0m + y_0$ and $b = -x_1m + y_1$

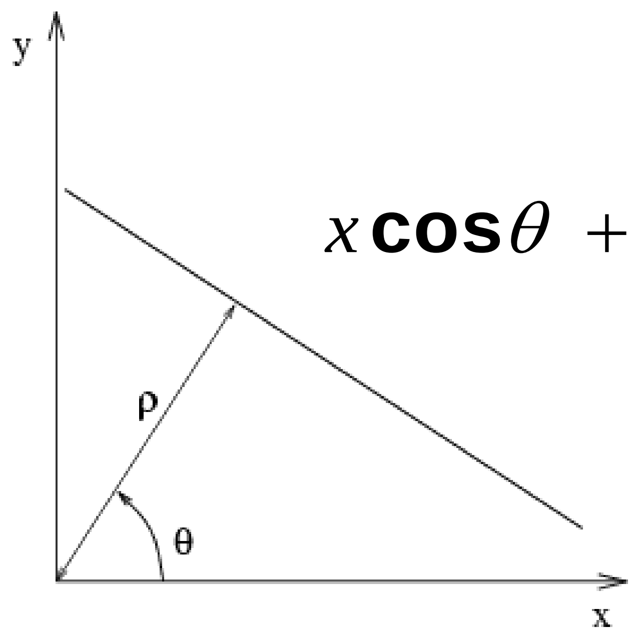


Parameter space representation

- Problems with the (m,b) space:
  - Unbounded parameter domain
  - Vertical lines require infinite m

Parameter space representation

- Problems with the (m,b) space:
  - Unbounded parameter domain
  - Vertical lines require infinite m
- Alternative: polar representation

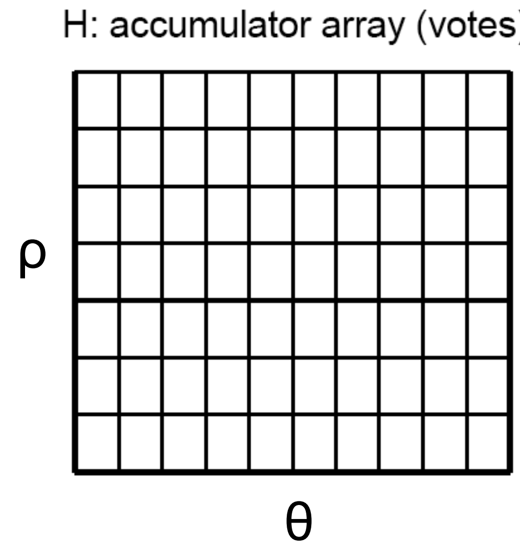


$$x\,\mathbf{cos}\theta + y\,\mathbf{sin}\theta = \rho$$

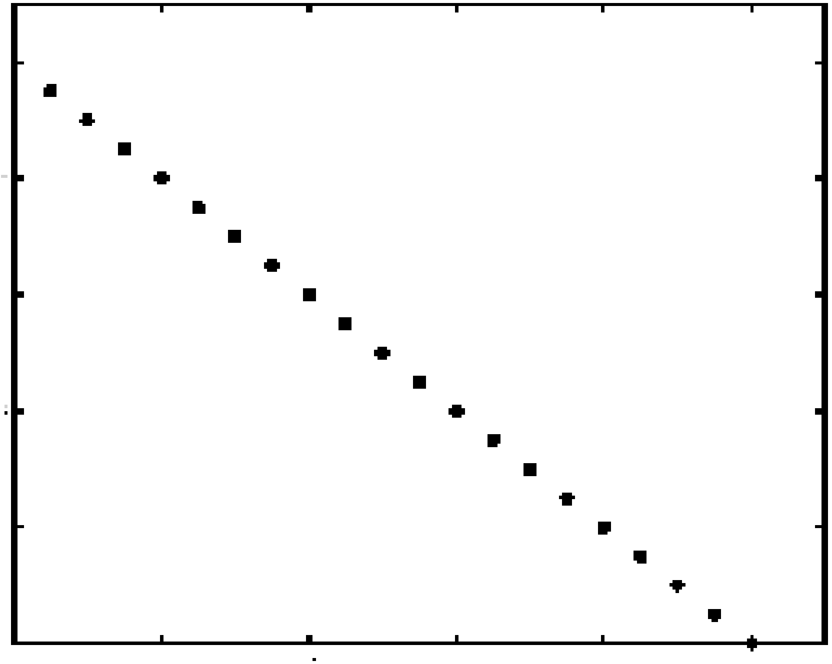
Each point will add a sinusoid in the (θ,ρ) parameter space

Algorithm outline

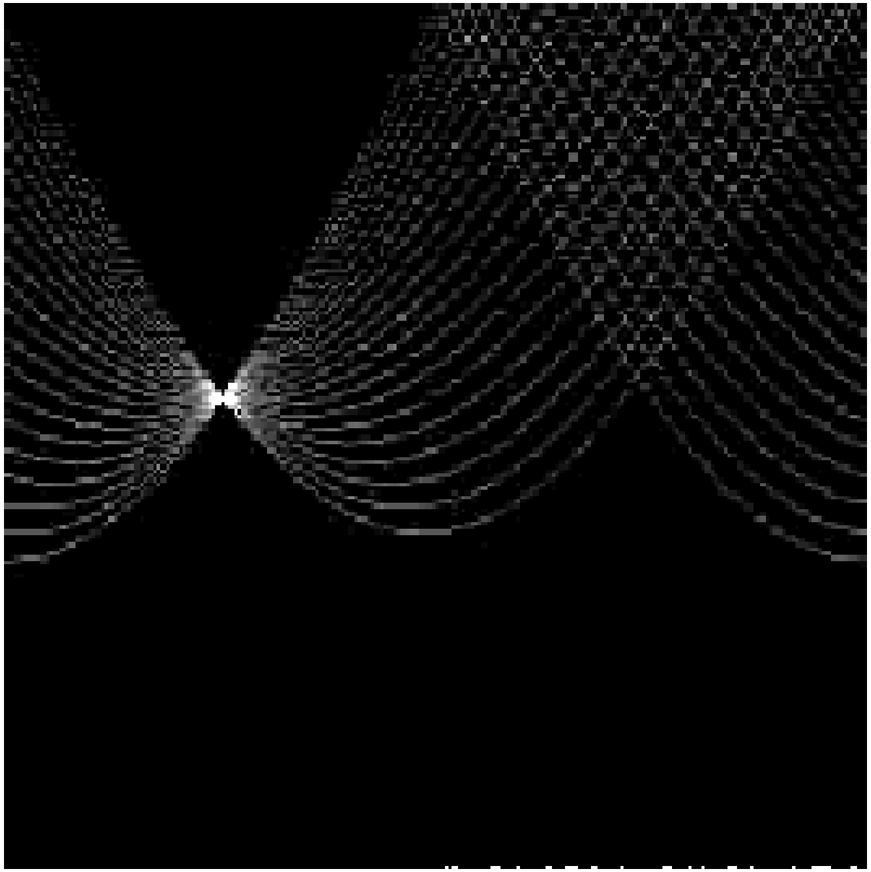
- Initialize accumulator H to all zeros
- For each edge point (x,y) in the image
  For θ = 0 to 180
    $\rho = x \cos \theta + y \sin \theta$
    $H(\theta, \rho) = H(\theta, \rho) + 1$
  end
  end
- Find the value(s) of (θ, ρ) where $H(\theta, \rho)$ is a local maximum
  - The detected line in the image is given by $\rho = x \cos \theta + y \sin \theta$



Basic illustration



features

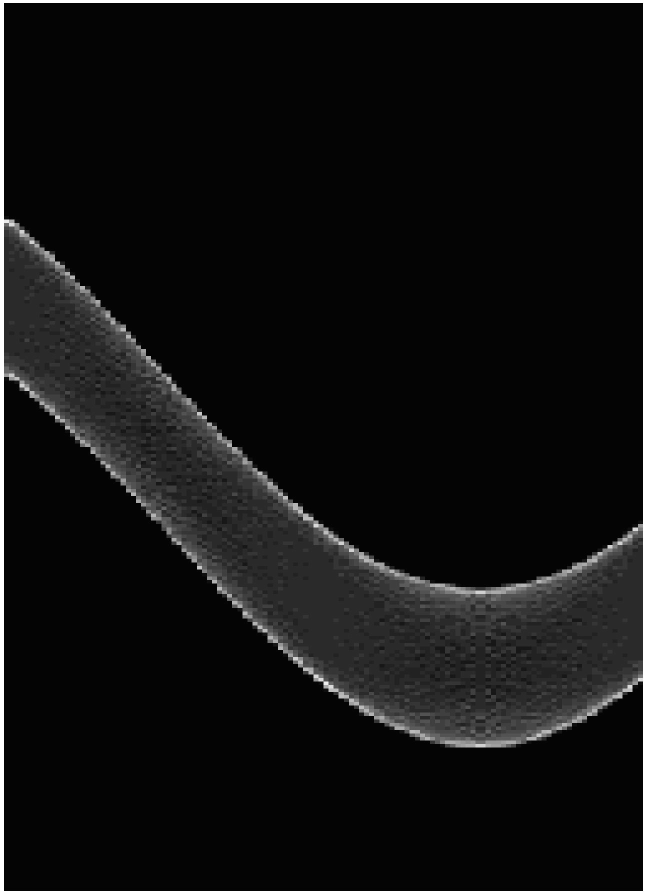


votes

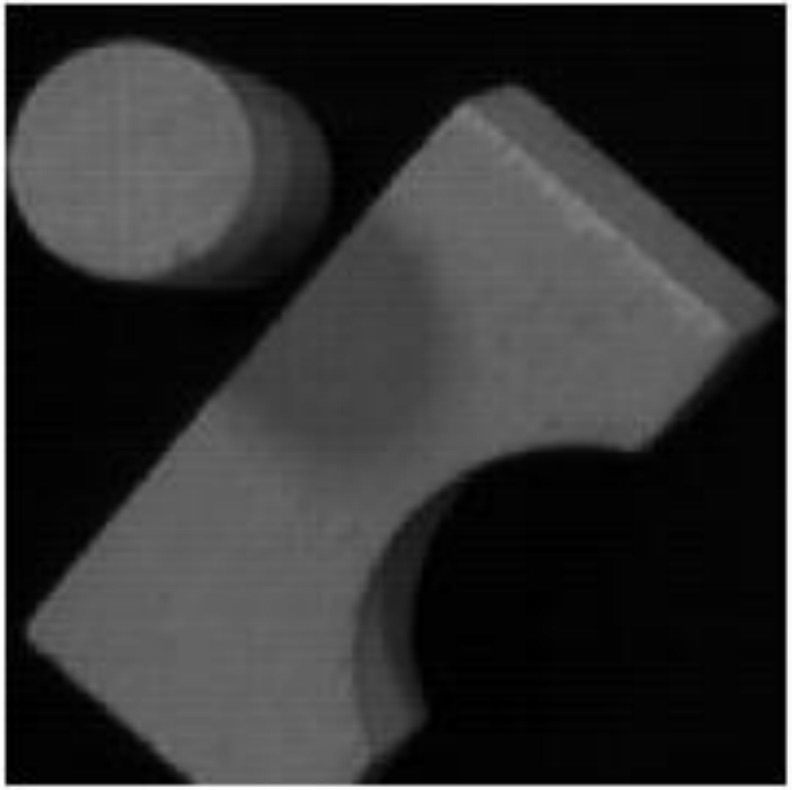
Other shapes

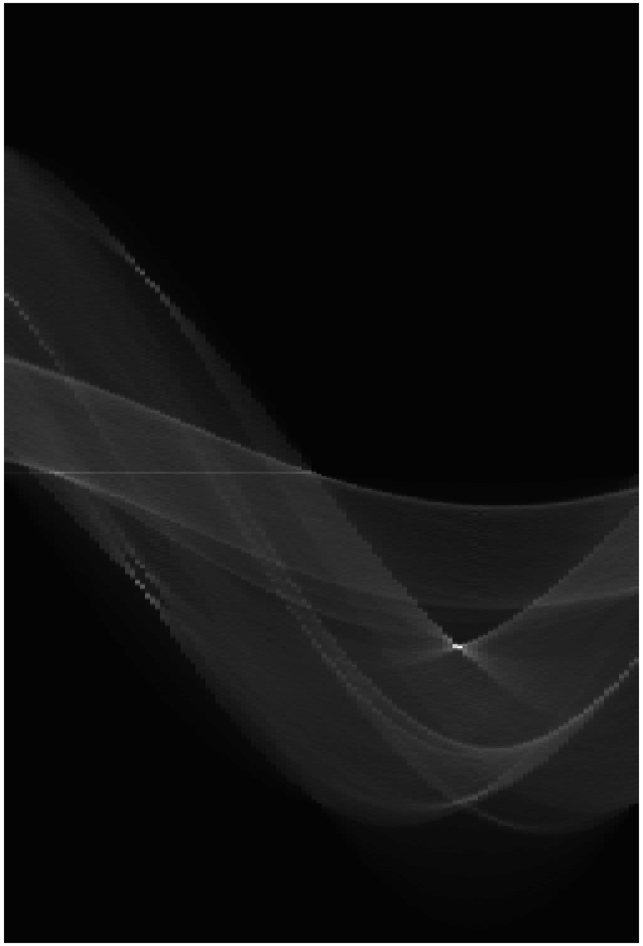
Square

Circle

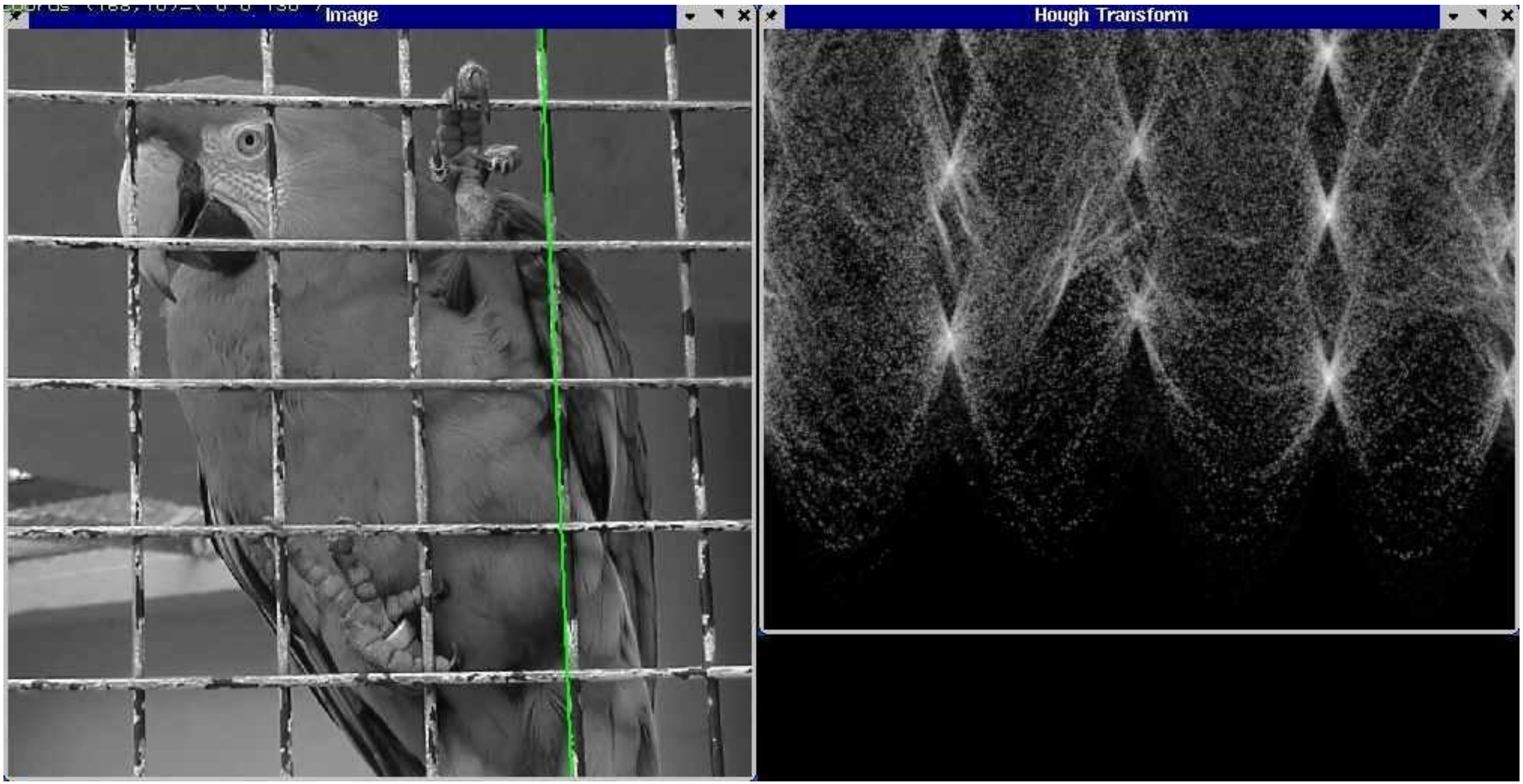


Several lines



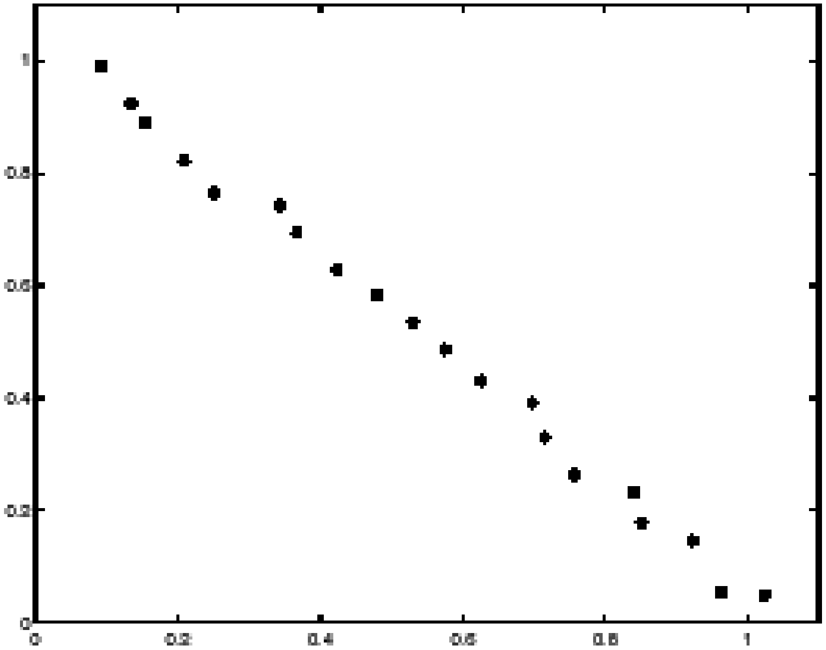


A more complicated image



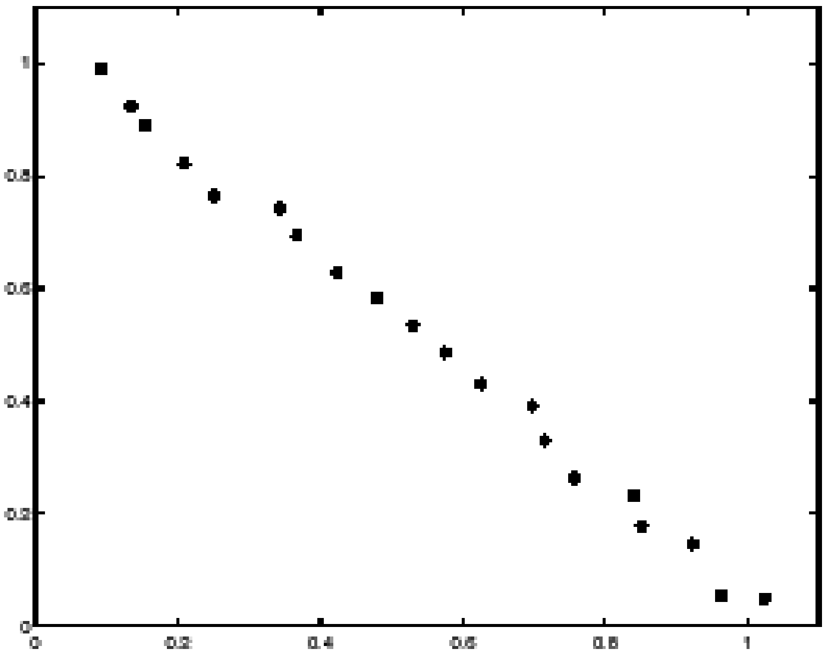
http://ostatic.com/files/images/ss_hough.jpg

Effect of noise

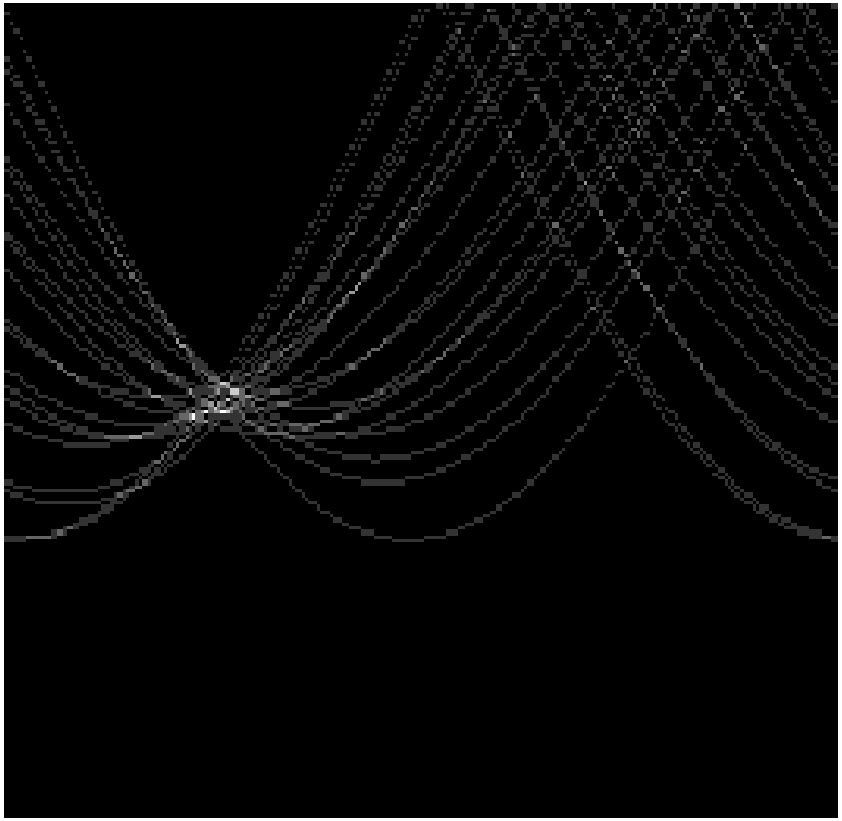


features

Effect of noise



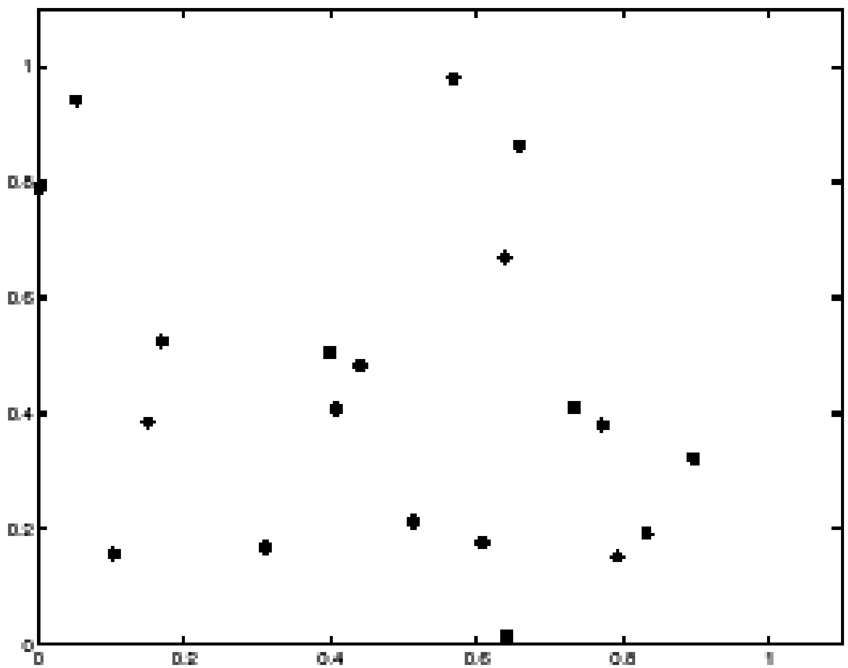
features



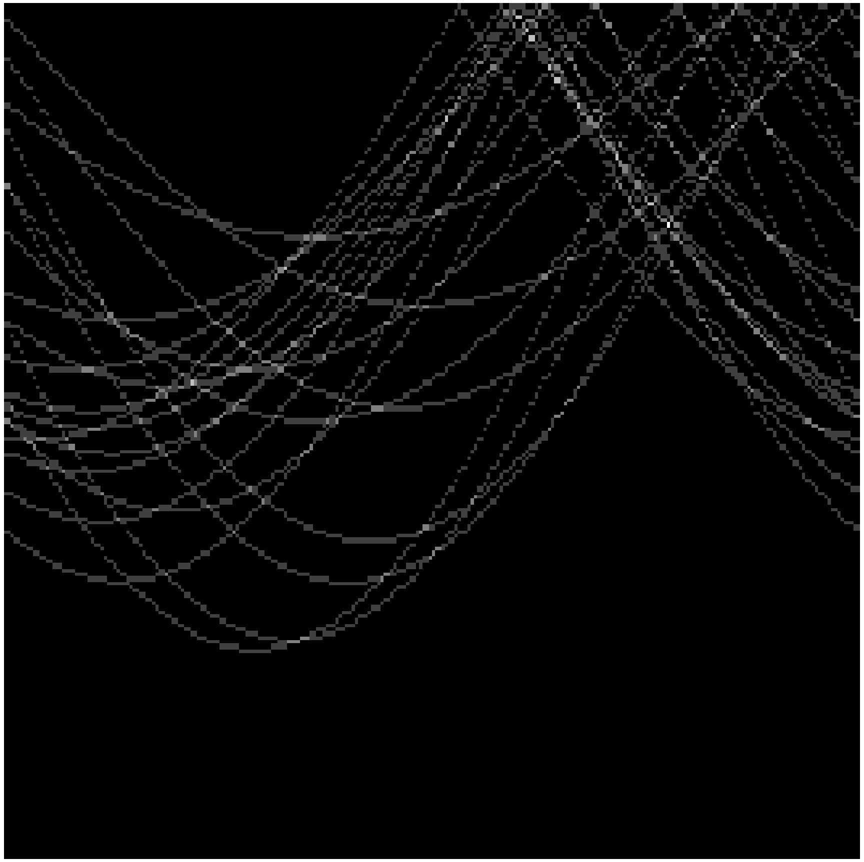
votes

Peak gets fuzzy and hard to locate

Random points



features



votes

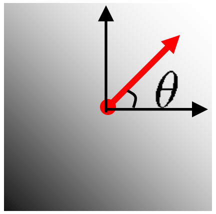
Uniform noise can lead to spurious peaks in the array

Dealing with noise

- Choose a good grid / discretization
  - Too coarse: large votes obtained when too many different lines correspond to a single bucket
  - Too fine: miss lines because some points that are not exactly collinear cast votes for different buckets
- Increment neighboring bins (smoothing in accumulator array)
- Try to get rid of irrelevant features
  - Take only edge points with significant gradient magnitude

Incorporating image gradients

- Recall: when we detect an edge point, we also know its gradient direction
- But this means that the line is uniquely determined!



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

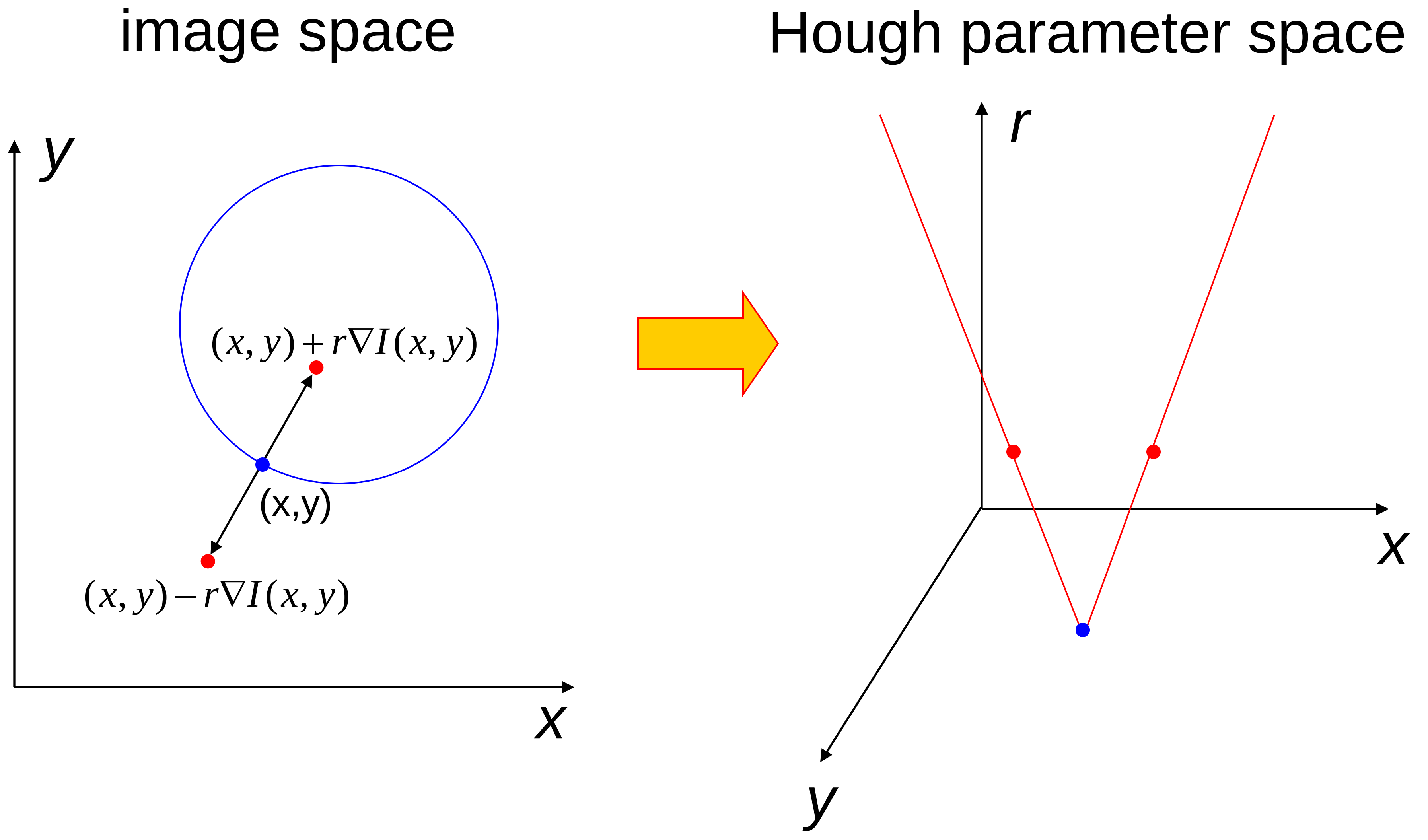
- Modified Hough transform:

For each edge point (x,y)
    θ = gradient orientation at (x,y)
    ρ = x cos θ + y sin θ
    H(θ, ρ) = H(θ, ρ) + 1
end

Hough transform for circles

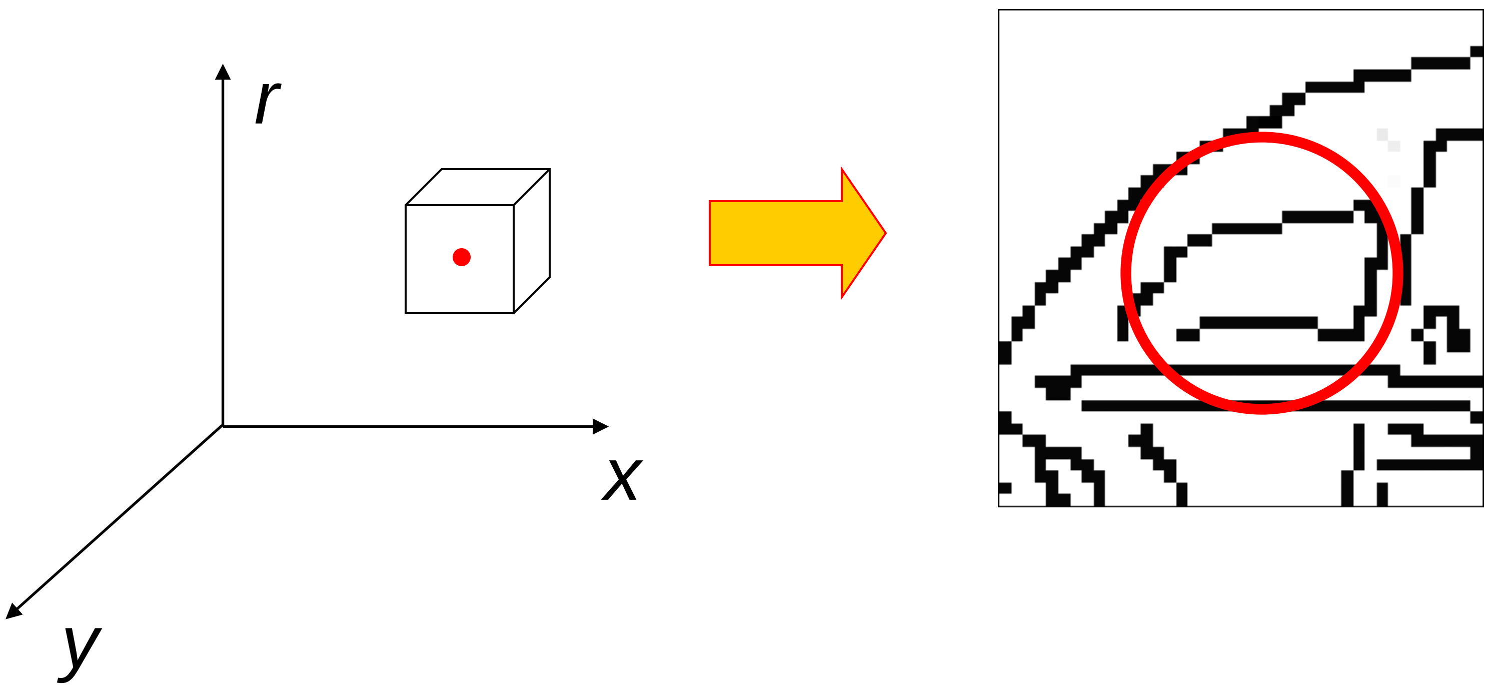
- How many dimensions will the parameter space have?
- Given an oriented edge point, what are all possible bins that it can vote for?

Hough transform for circles



Hough transform for circles

- Conceptually equivalent procedure: for each (x,y,r), draw the corresponding circle in the image and compute its "support"



Is this more or less efficient than voting with features?

Finding straight lines

- Another solution: get connected components of pixels and check for straightness

Finding line segments using connected components

1. Compute canny edges
   – Compute: gx, gy (DoG in x,y directions)
   – Compute: theta = atan(gy / gx)
2. Assign each edge to one of 8 directions
3. For each direction d, get edgelets:
   – find connected components for edge pixels with directions in {d-1, d, d+1}
4. Compute straightness and theta of edgelets using eig of x,y 2nd moment matrix of their points

$$\mathbf{M} = \begin{bmatrix} \sum (x-\mu_x)^2 & \sum (x-\mu_x)(y-\mu_y) \\ \sum (x-\mu_x)(y-\mu_y) & \sum (y-\mu_y)^2 \end{bmatrix} \qquad [v,\lambda] = \mathrm{eig}(\mathbf{M})$$

Larger eigenvector

$$\theta = \mathrm{atan}\,2(v(2,2), v(1,2))$$

$$conf = \lambda_2 / \lambda_1$$

5. Threshold on straightness, store segment

Slides from Derek Hoiem

1. Image → Canny

2. Canny lines → … →straight edges

Comparison

Hough Transform Method

Connected Components Method

Things to remember

- Canny edge detector = smooth → derivative → thin → threshold → link

- Generalized Hough transform = points vote for shape parameters

- Straight line detector = canny + gradient orientations → orientation binning → linking → check for straightness

Next classes

- Generalized Hough Transform

- Fitting and Registration

- EM (mixture models)

Questions