

# Feature Matching and Robust Fitting

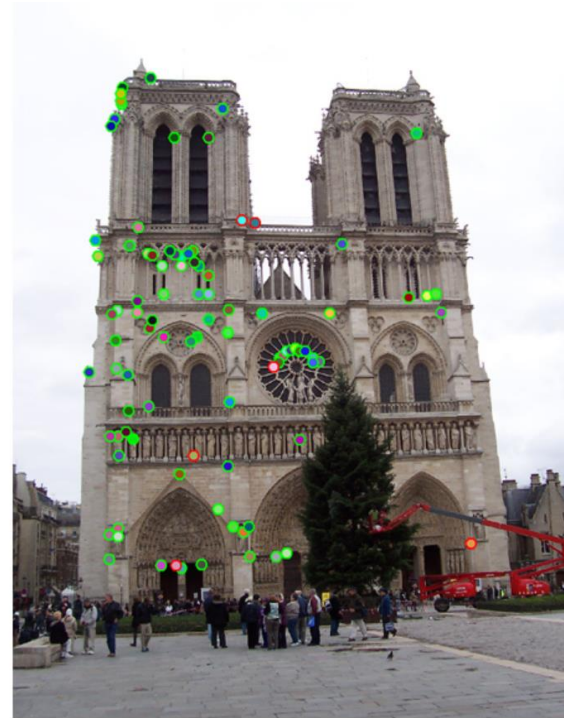
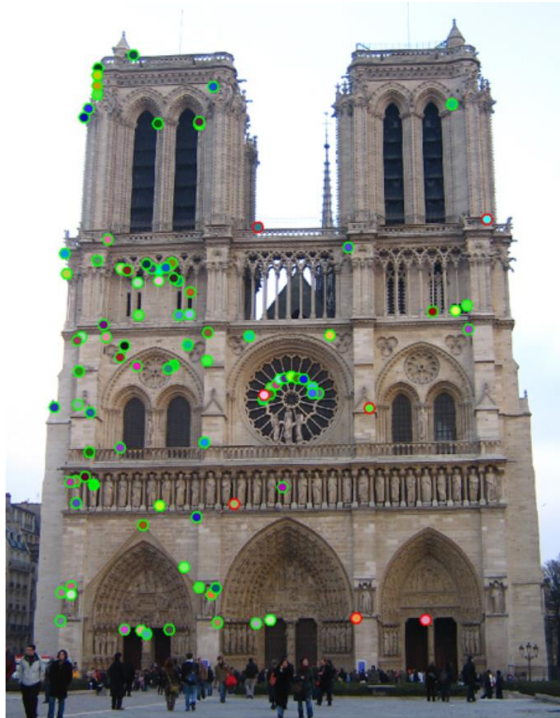
Read Szeliski 4.1

Computer Vision

CS 143, Brown

James Hays

# Project 2 questions?



The top 100 most confident local feature matches from a baseline implementation of project 2. In this case, 93 were correct (highlighted in green) and 7 were incorrect (highlighted in red).

## Project 2: Local Feature Matching

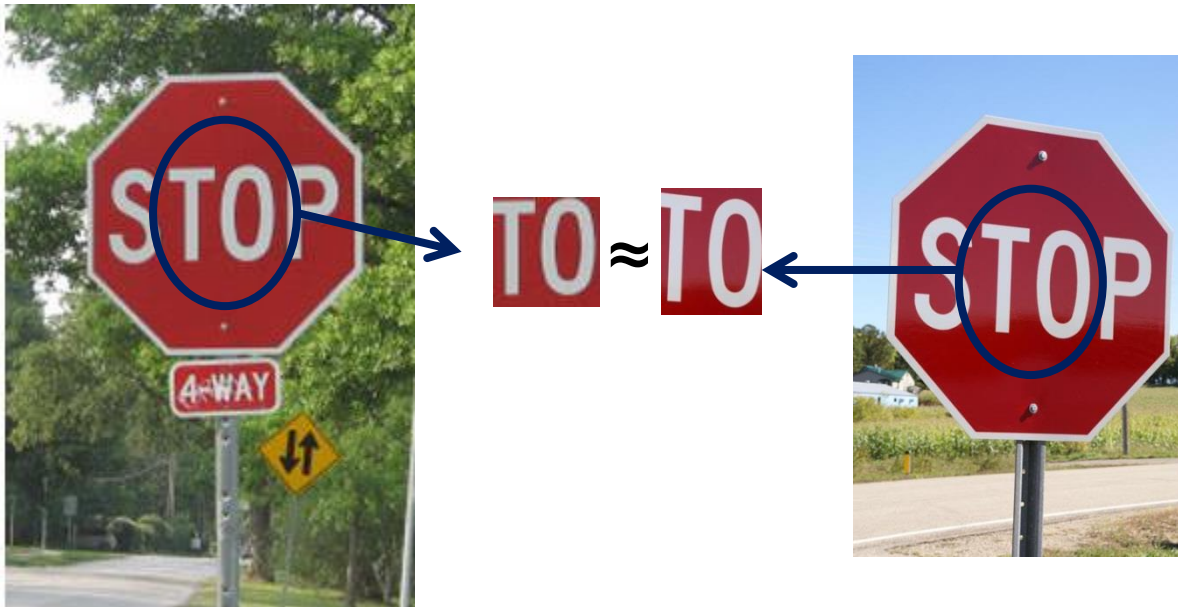
### CS 143: Introduction to Computer Vision

#### Brief

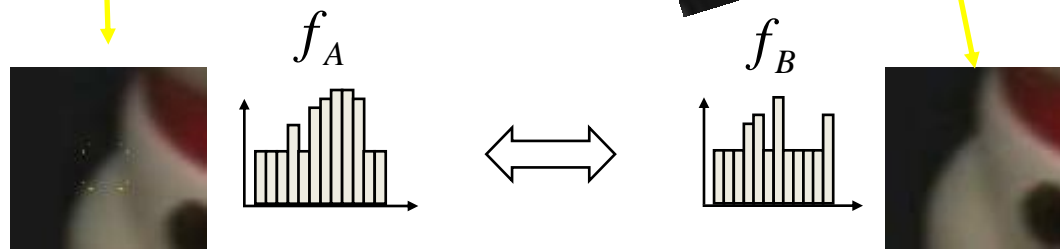
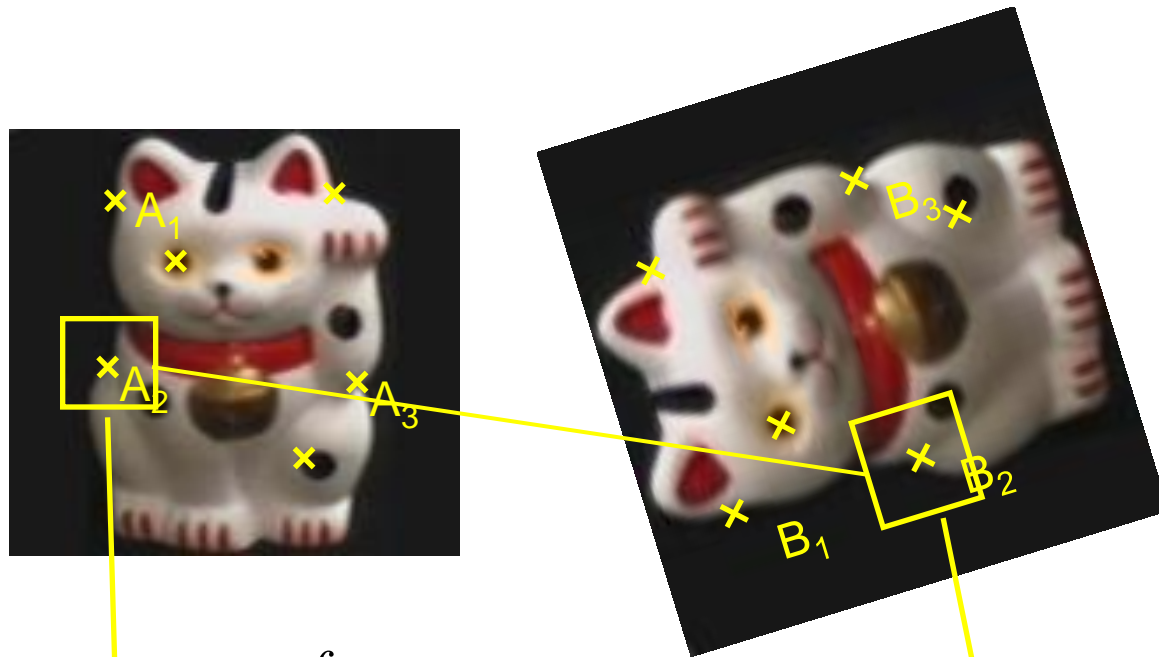
- Due: 11:59pm on Monday, October 7th, 2013
- Stencil code: `/course/cs143/asgn/proj2/code/`
- Data: `/course/cs143/asgn/proj2/data/` includes 93 images from 9 different outdoor scenes.
- Html writeup template: `/course/cs143/asgn/proj2/html/`
- *Partial* project materials are also available in **proj2.zip (1.7 MB)**. Includes only the two test images shown above.
- Handin: `cs143_handin proj2`
- Required files: `README, code/, html/, html/index.html`

# This section: correspondence and alignment

- Correspondence: matching points, patches, edges, or regions across images



# Overview of Keypoint Matching

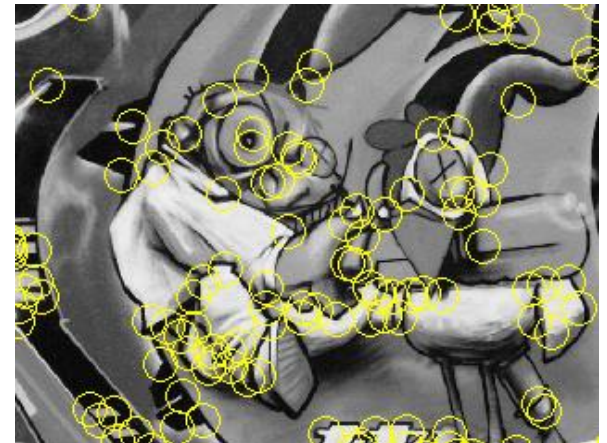


$$d(f_A, f_B) < T$$

1. Find a set of distinctive keypoints
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

# Review: Interest points

- Keypoint detection: repeatable and distinctive
  - Corners, blobs, stable regions
  - Harris, DoG, MSER

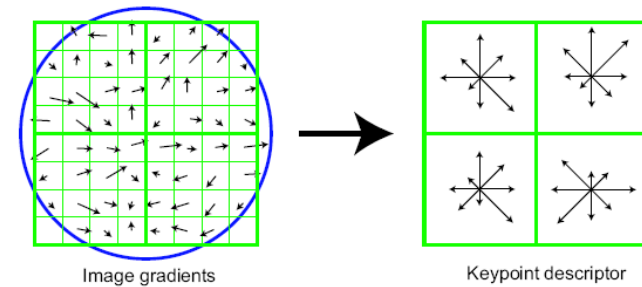


# Review: Choosing an interest point detector

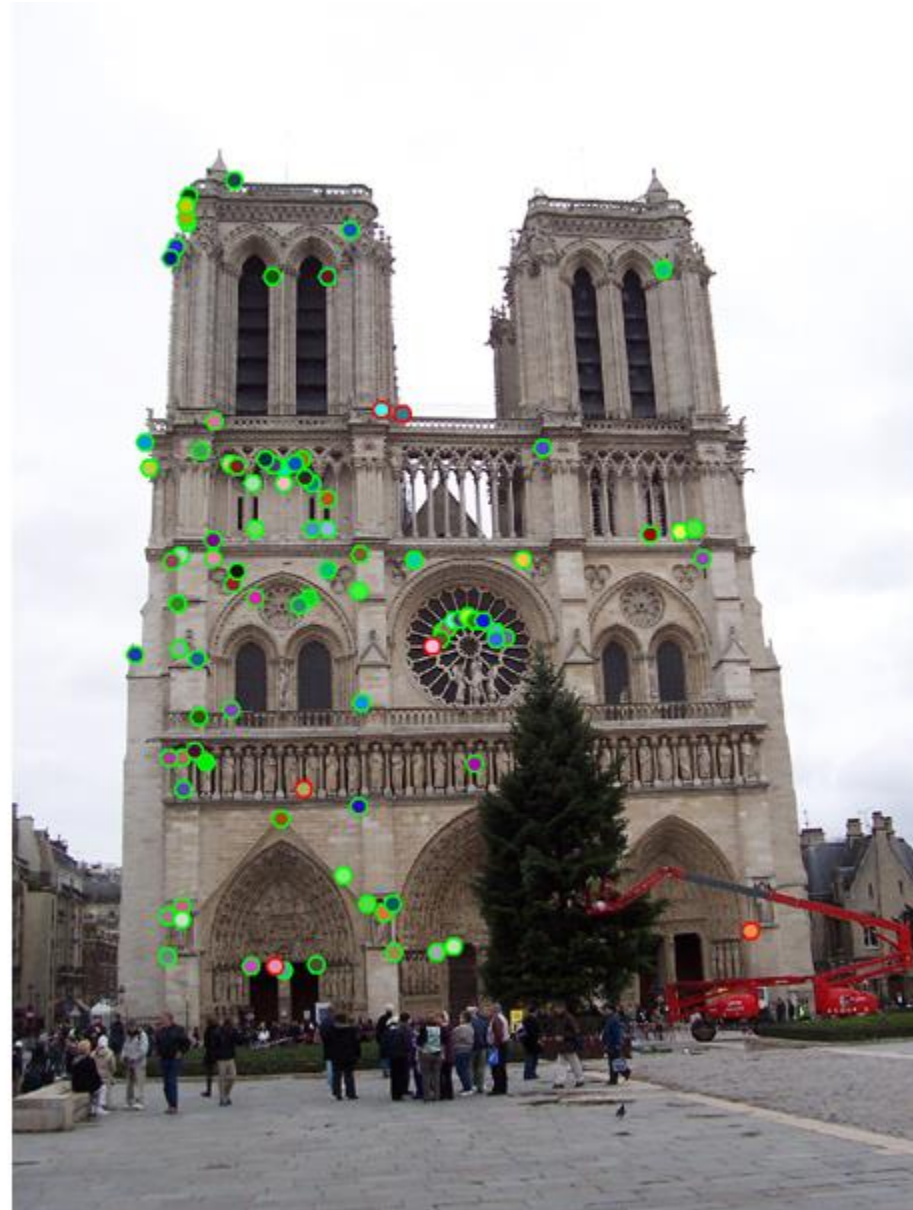
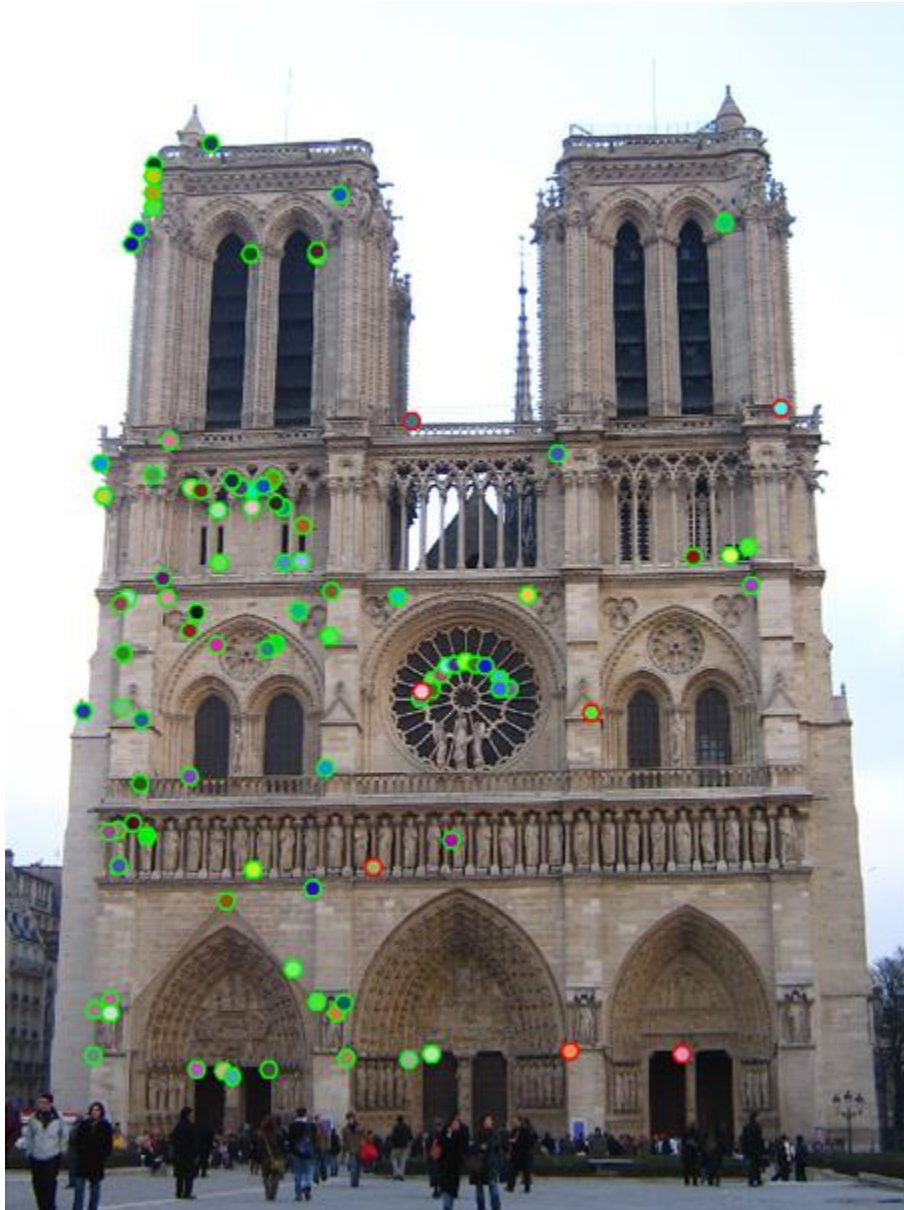
- What do you want it for?
  - Precise localization in x-y: Harris
  - Good localization in scale: Difference of Gaussian
  - Flexible region shape: MSER
- Best choice often application dependent
  - Harris-/Hessian-Laplace/DoG work well for many natural categories
  - MSER works well for buildings and printed things
- Why choose?
  - Get more points with more detectors
- There have been extensive evaluations/comparisons
  - [Mikolajczyk et al., IJCV'05, PAMI'05]
  - All detectors/descriptors shown here work well

# Review: Local Descriptors

- Most features can be thought of as templates, histograms (counts), or combinations
- The ideal descriptor should be
  - Robust and Distinctive
  - Compact and Efficient
- Most available descriptors focus on edge/gradient information
  - Capture texture information
  - Color rarely used



# How do we decide which features match?





# Feature Matching

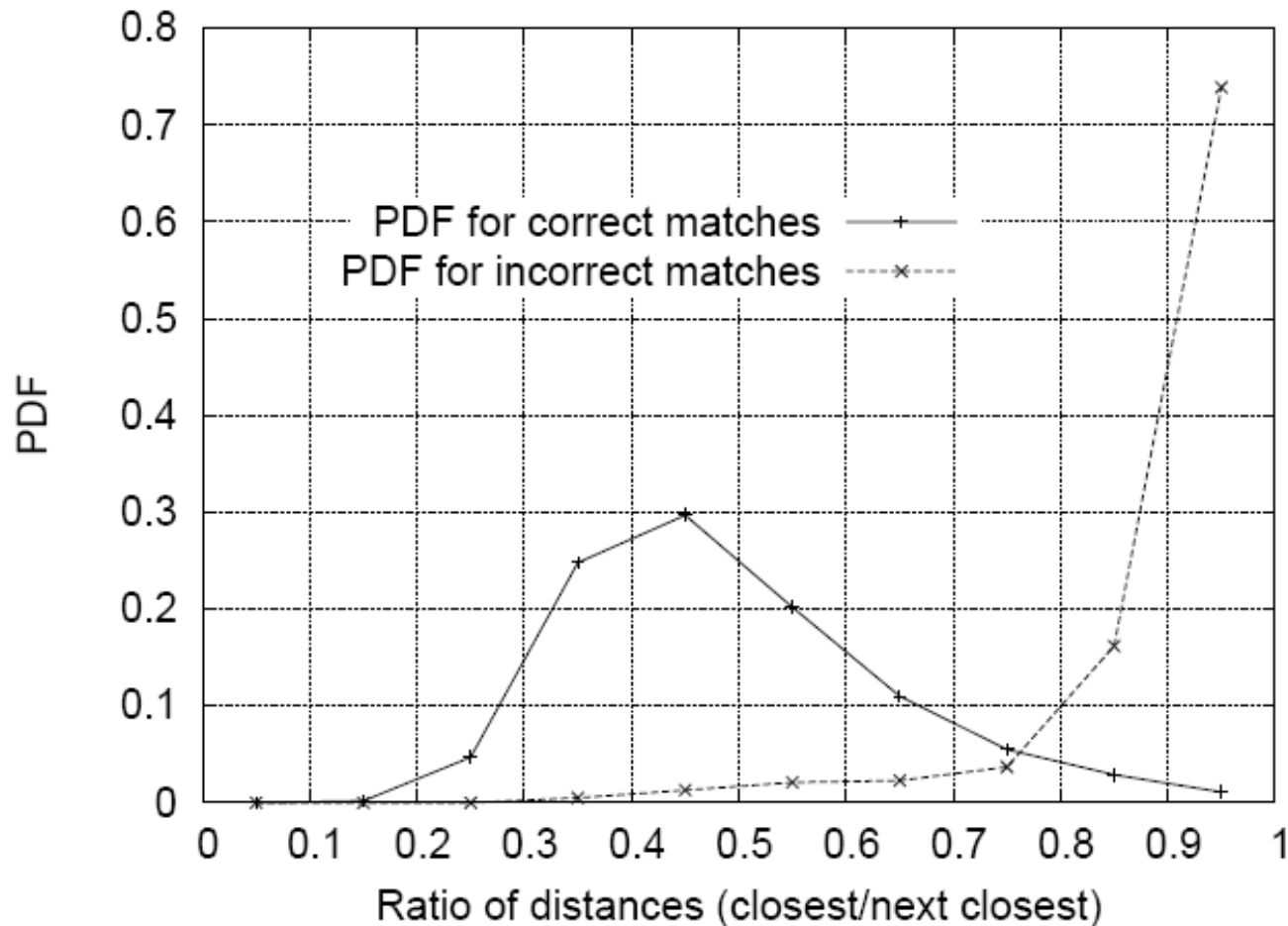
- Szeliski 4.1.3
  - Simple feature-space methods
  - Evaluation methods
  - Acceleration methods
  - Geometric verification (Chapter 6)

# Feature Matching

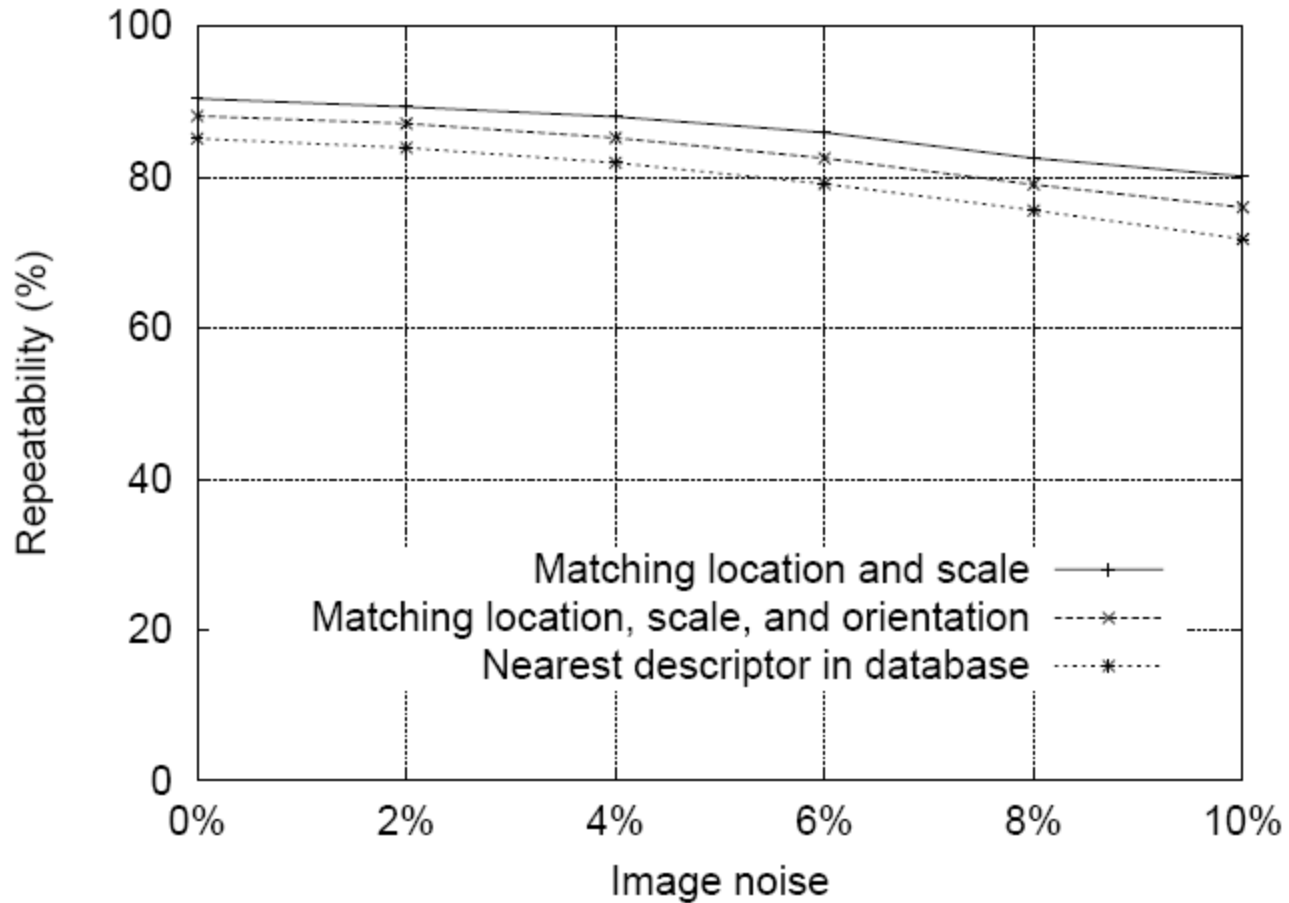
- Simple criteria: One feature matches to another if those features are nearest neighbors and their distance is below some threshold.
- Problems:
  - Threshold is difficult to set
  - Non-distinctive features could have lots of close matches, only one of which is correct

# Matching Local Features

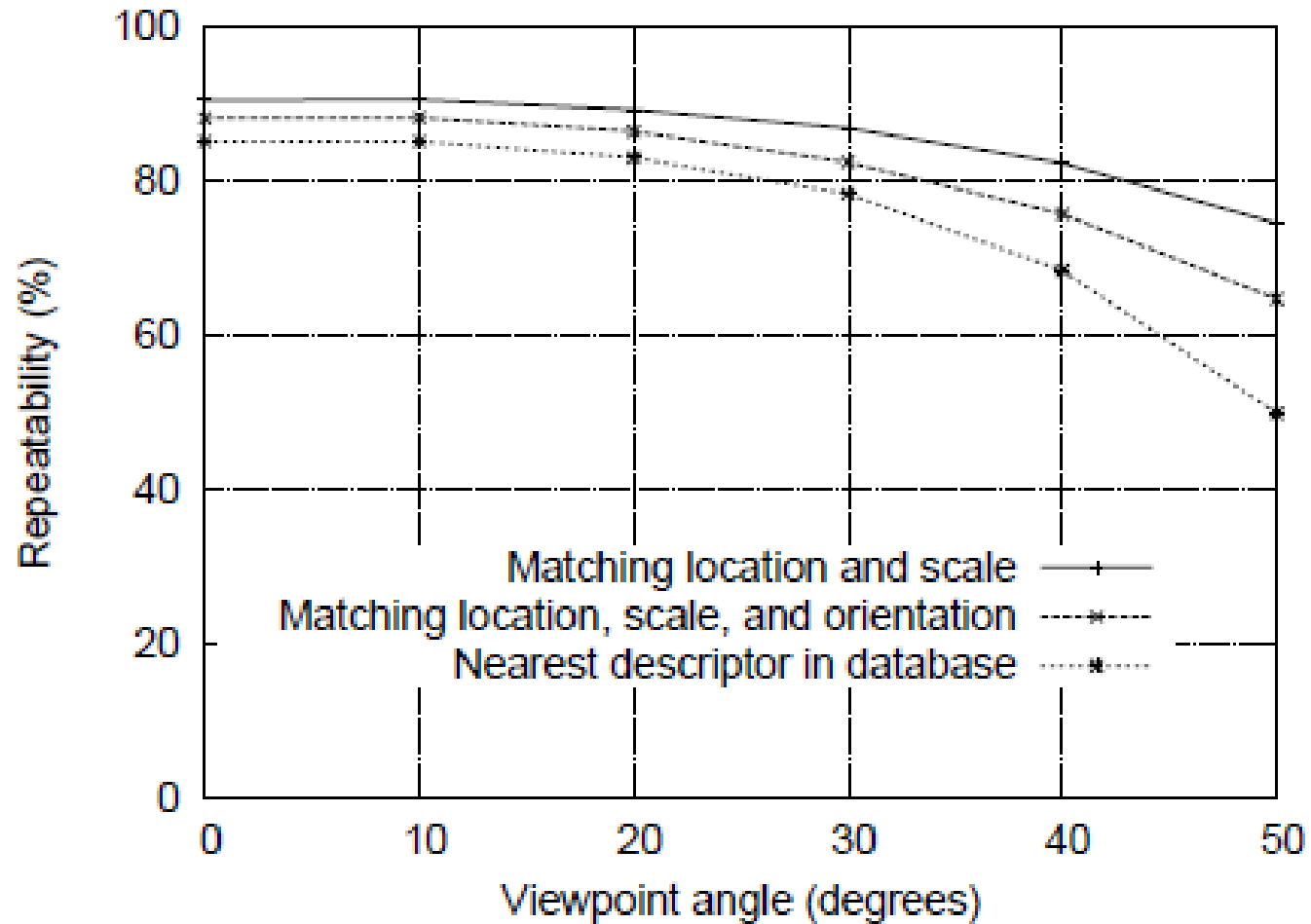
- Threshold based on the ratio of 1<sup>st</sup> nearest neighbor to 2<sup>nd</sup> nearest neighbor distance.



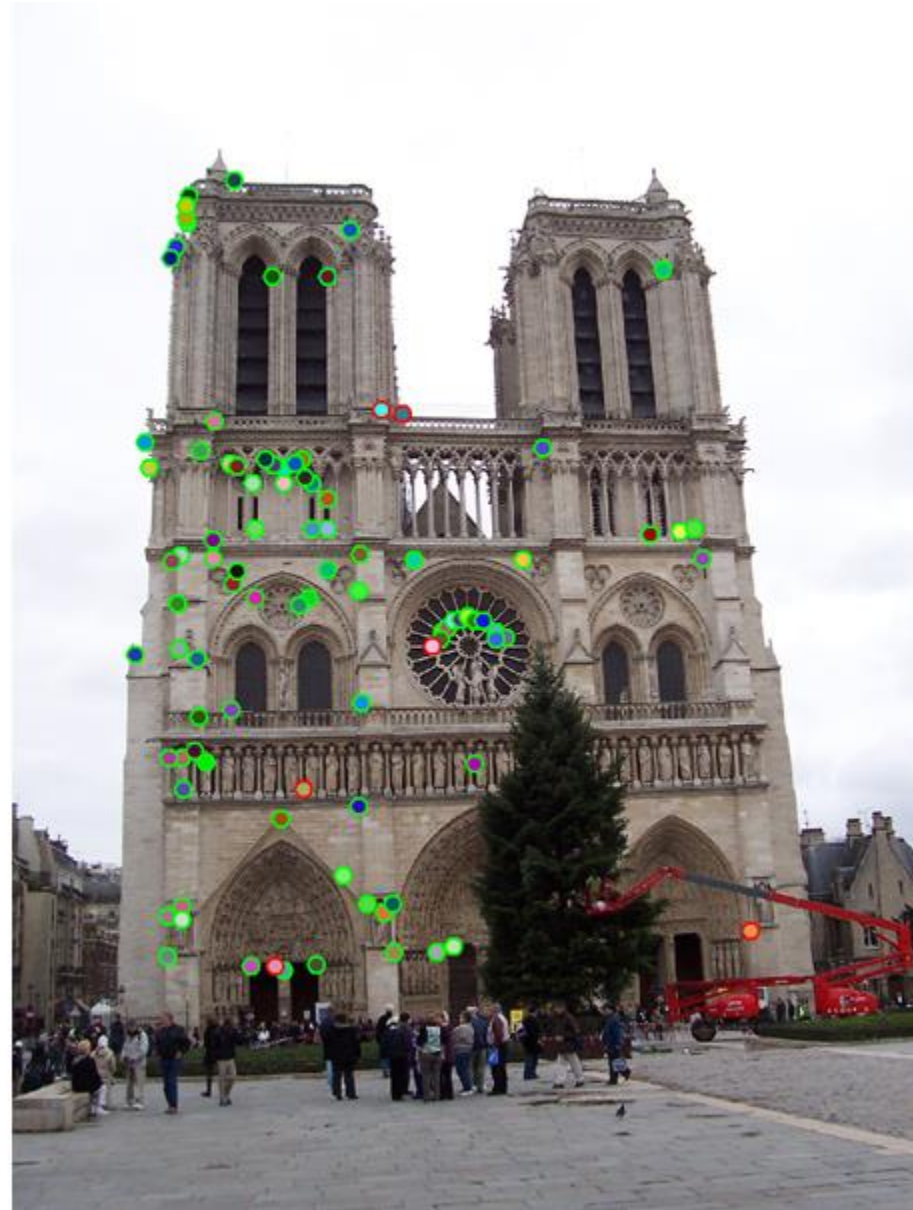
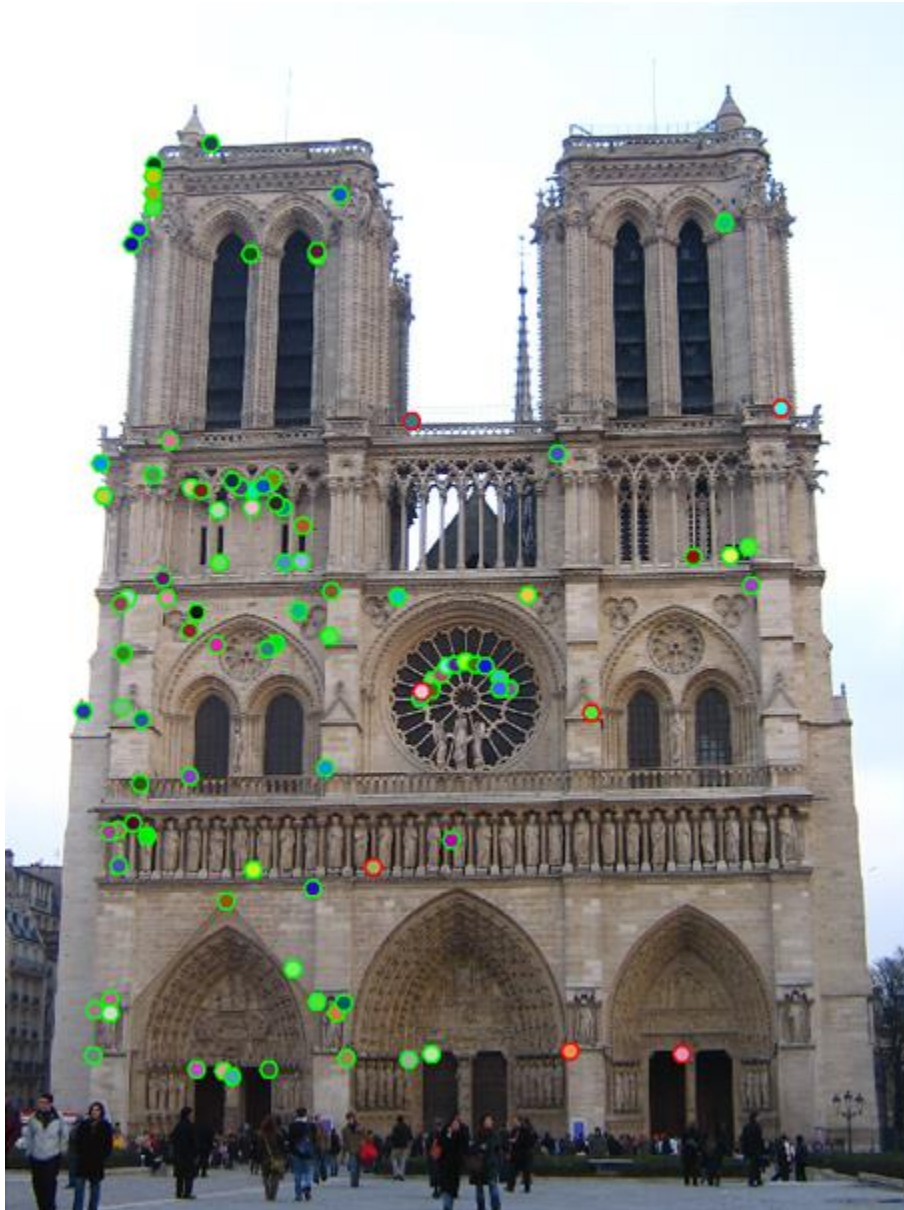
# SIFT Repeatability



# SIFT Repeatability



# How do we decide which features match?



Fitting: find the parameters of a model that best fit the data

Alignment: find the parameters of the transformation that best align matched points

# Fitting and Alignment

- Design challenges
  - Design a suitable **goodness of fit** measure
    - Similarity should reflect application goals
    - Encode robustness to outliers and noise
  - Design an **optimization** method
    - Avoid local optima
    - Find best parameters quickly



# Fitting and Alignment: Methods

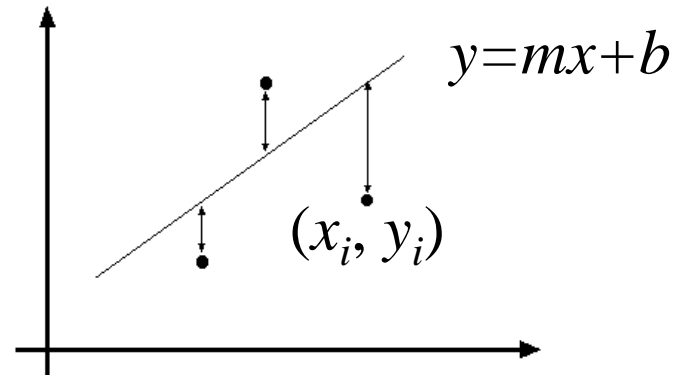
- Global optimization / Search for parameters
  - Least squares fit
  - Robust least squares
  - Iterative closest point (ICP)
- Hypothesize and test
  - Generalized Hough transform
  - RANSAC

# Simple example: Fitting a line

# Least squares line fitting

- Data:  $(x_1, y_1), \dots, (x_n, y_n)$
- Line equation:  $y_i = mx_i + b$
- Find  $(m, b)$  to minimize

$$E = \sum_{i=1}^n (y_i - mx_i - b)^2$$



$$E = \sum_{i=1}^n \left( \begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - y_i \right)^2 = \left\| \begin{bmatrix} x_1 \\ \vdots \\ x_n \\ 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right\|^2 = \|\mathbf{A}\mathbf{p} - \mathbf{y}\|^2$$

$$= \mathbf{y}^T \mathbf{y} - 2(\mathbf{A}\mathbf{p})^T \mathbf{y} + (\mathbf{A}\mathbf{p})^T (\mathbf{A}\mathbf{p})$$

$$\frac{dE}{dp} = 2\mathbf{A}^T \mathbf{A}\mathbf{p} - 2\mathbf{A}^T \mathbf{y} = 0$$

Matlab:  $\mathbf{p} = \mathbf{A} \setminus \mathbf{y};$

$$\mathbf{A}^T \mathbf{A}\mathbf{p} = \mathbf{A}^T \mathbf{y} \Rightarrow \mathbf{p} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{y}$$

# Least squares (global) optimization

## Good

- Clearly specified objective
- Optimization is easy

## Bad

- May not be what you want to optimize
- Sensitive to outliers
  - Bad matches, extra points
- Doesn't allow you to get multiple good fits
  - Detecting multiple objects, lines, etc.

# Robust least squares (to deal with outliers)

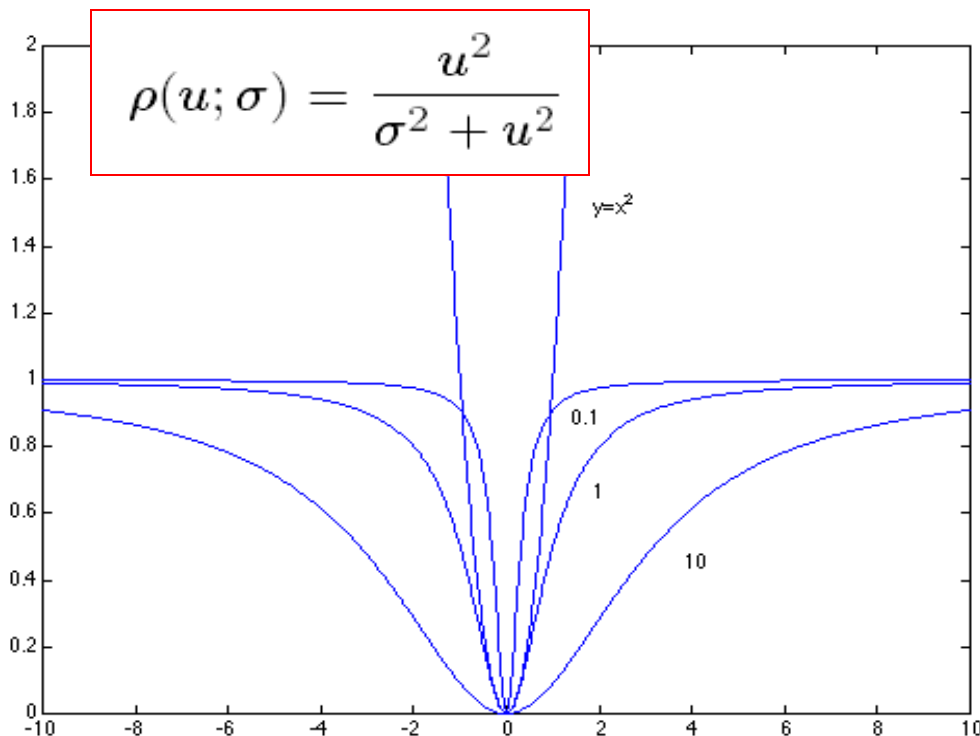
General approach:

minimize

$$\sum_i \rho(u_i(x_i, \theta); \sigma) \quad u^2 = \sum_{i=1}^n (y_i - mx_i - b)^2$$

$u_i(x_i, \theta)$  – residual of  $i^{\text{th}}$  point w.r.t. model parameters  $\vartheta$

$\rho$  – robust function with scale parameter  $\sigma$



## The robust function $\rho$

- Favors a configuration with small residuals
- Constant penalty for large residuals

# Robust Estimator

1. Initialize: e.g., choose  $\theta$  by least squares fit and  $\sigma = 1.5 \cdot \text{median}(\text{error})$

2. Choose params to minimize:  $\sum_i \frac{\text{error}(\theta, \text{data}_i)^2}{\sigma^2 + \text{error}(\theta, \text{data}_i)^2}$   
– E.g., numerical optimization

3. Compute new  $\sigma = 1.5 \cdot \text{median}(\text{error})$

4. Repeat (2) and (3) until convergence

# Other ways to search for parameters (for when no closed form solution exists)

- Line search
  1. For each parameter, step through values and choose value that gives best fit
  2. Repeat (1) until no parameter changes
- Grid search
  1. Propose several sets of parameters, evenly sampled in the joint set
  2. Choose best (or top few) and sample joint parameters around the current best; repeat
- Gradient descent
  1. Provide initial position (e.g., random)
  2. Locally search for better parameters by following gradient

# Hypothesize and test

1. Propose parameters
  - Try all possible
  - Each point votes for all consistent parameters
  - Repeatedly sample enough points to solve for parameters
2. Score the given parameters
  - Number of consistent points, possibly weighted by distance
3. Choose from among the set of parameters
  - Global or local maximum of scores
4. Possibly refine parameters using inliers



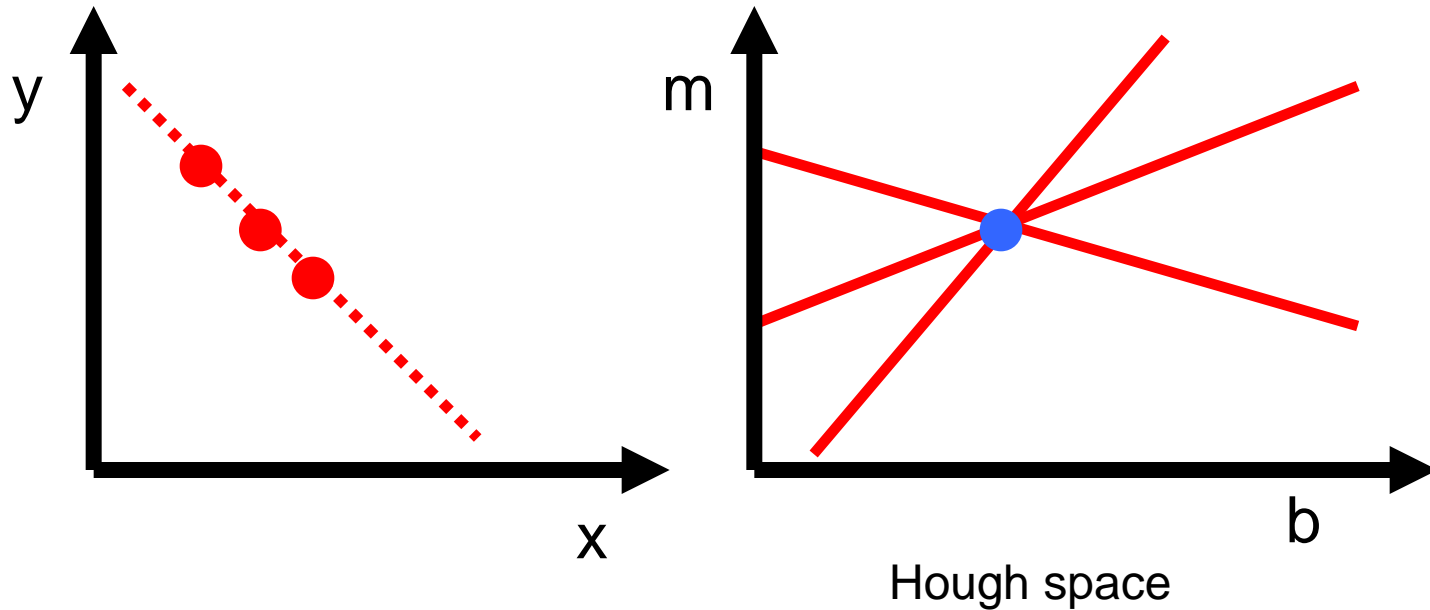
# Hough Transform: Outline

1. Create a grid of parameter values
2. Each point votes for a set of parameters, incrementing those values in grid
3. Find maximum or local maxima in grid

# Hough transform

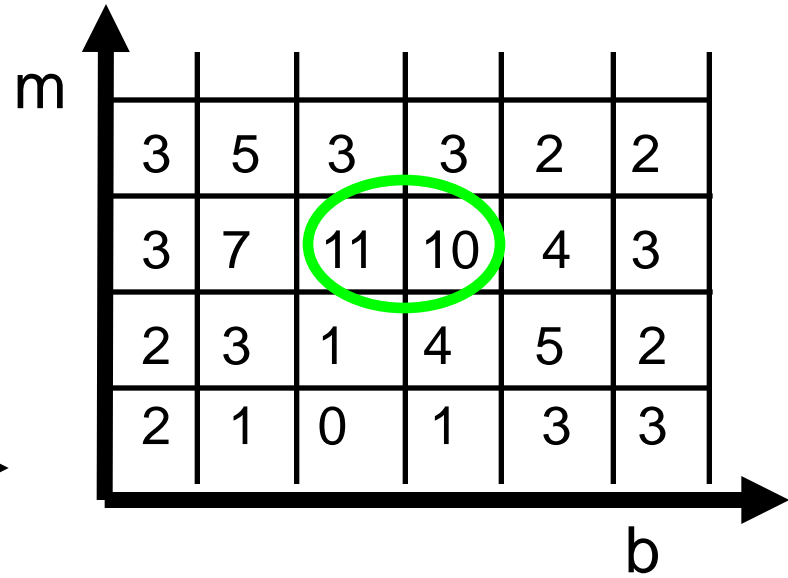
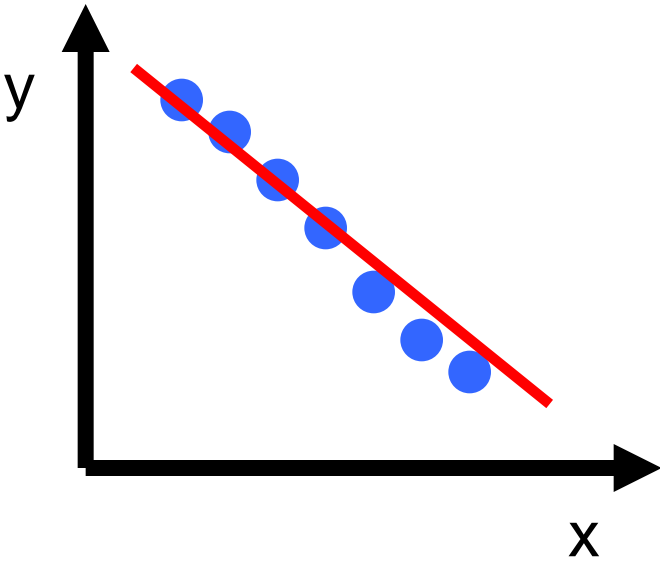
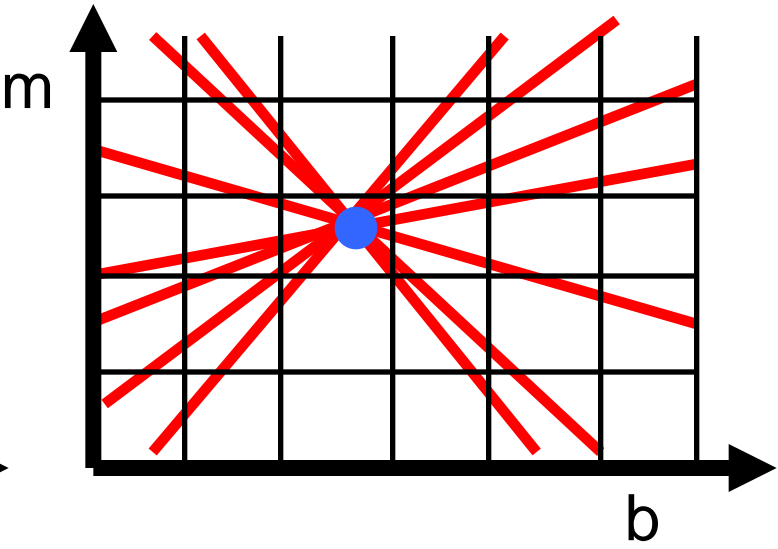
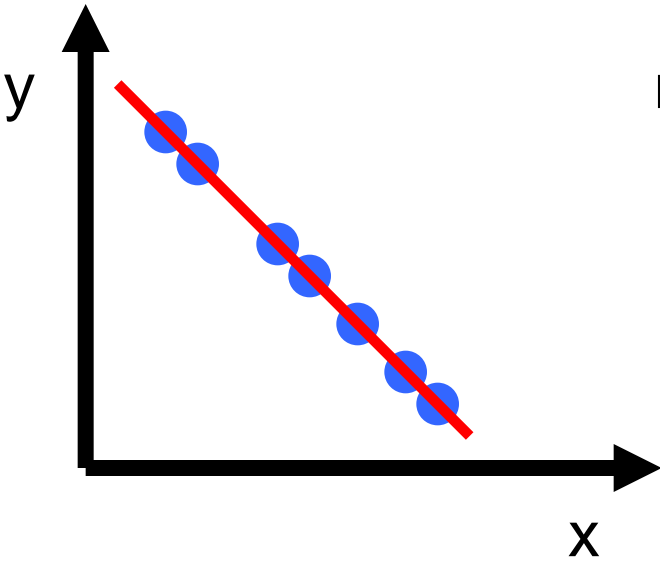
P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures*, Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Given a set of points, find the curve or line that explains the data points best



$$y = m x + b$$

# Hough transform

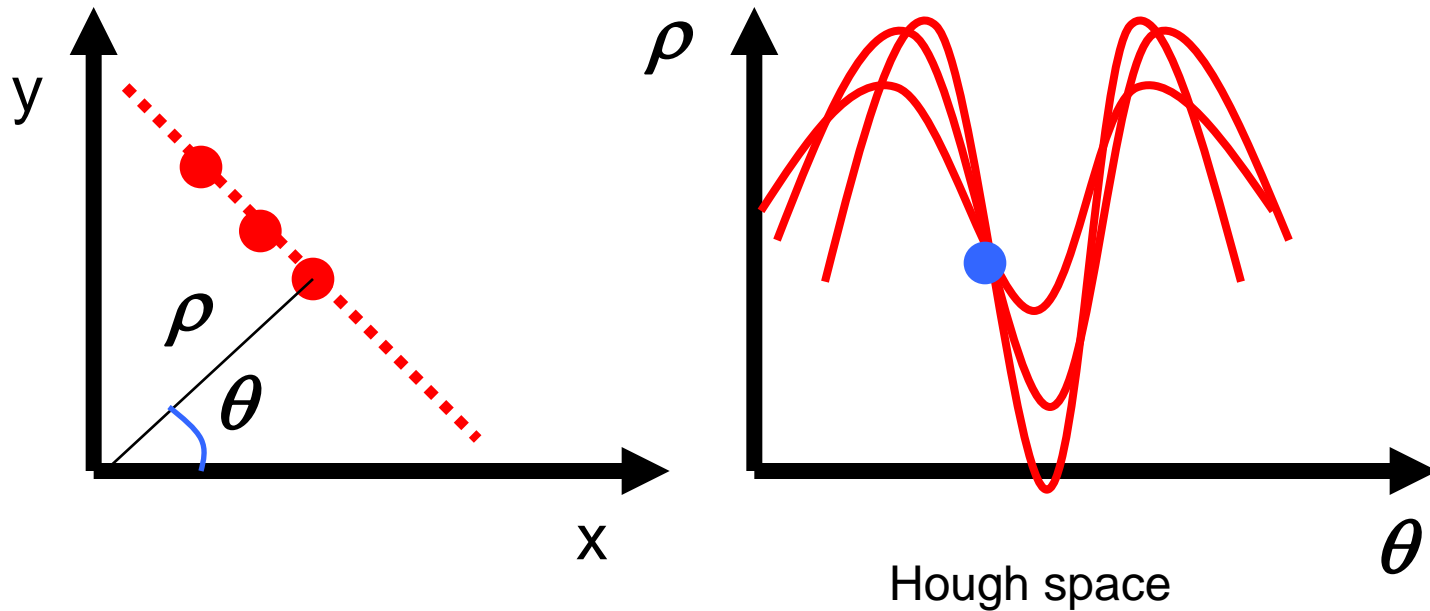


# Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures*, Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

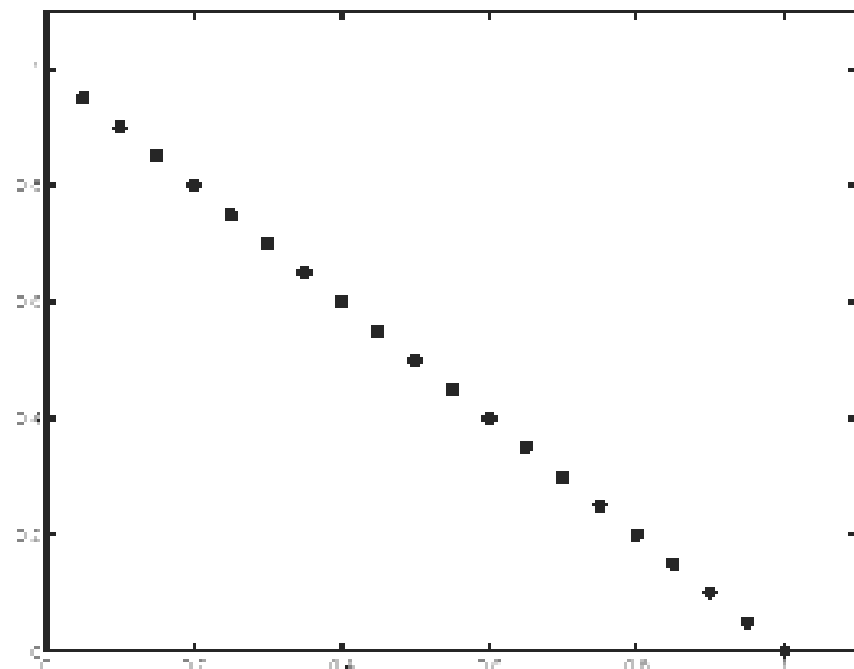
Issue : parameter space  $[m,b]$  is unbounded...

Use a polar representation for the parameter space

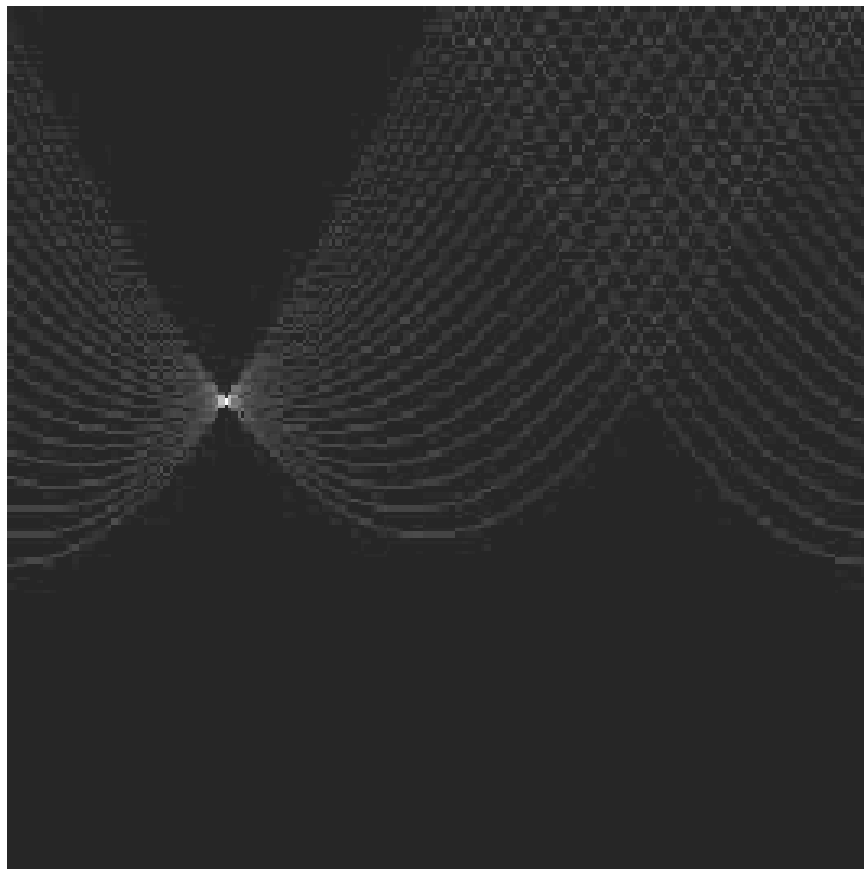


$$x \cos \theta + y \sin \theta = \rho$$

# Hough transform - experiments

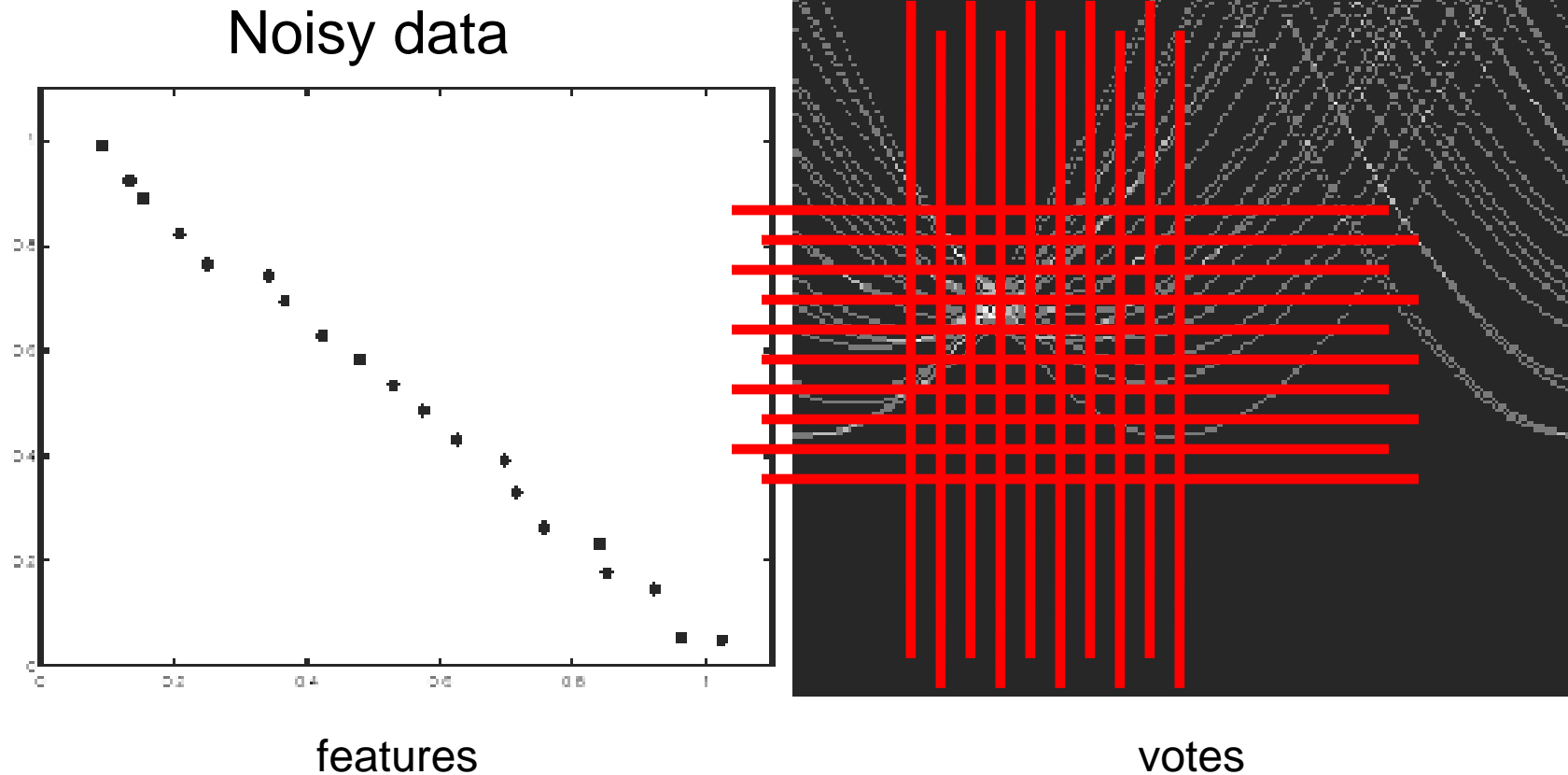


features



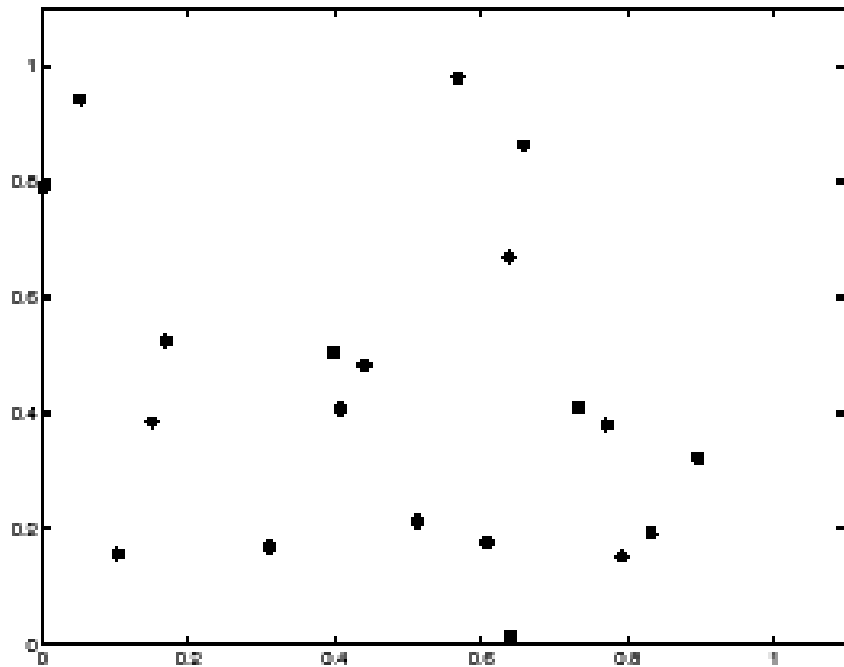
votes

# Hough transform - experiments

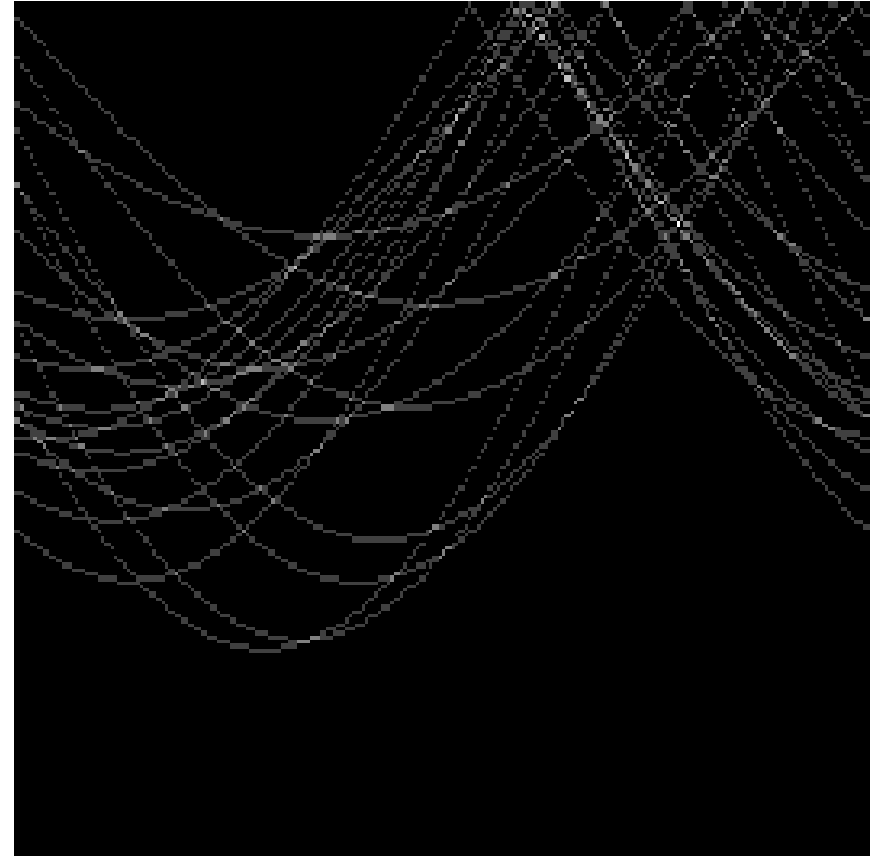


Need to adjust grid size or smooth

# Hough transform - experiments



features



votes

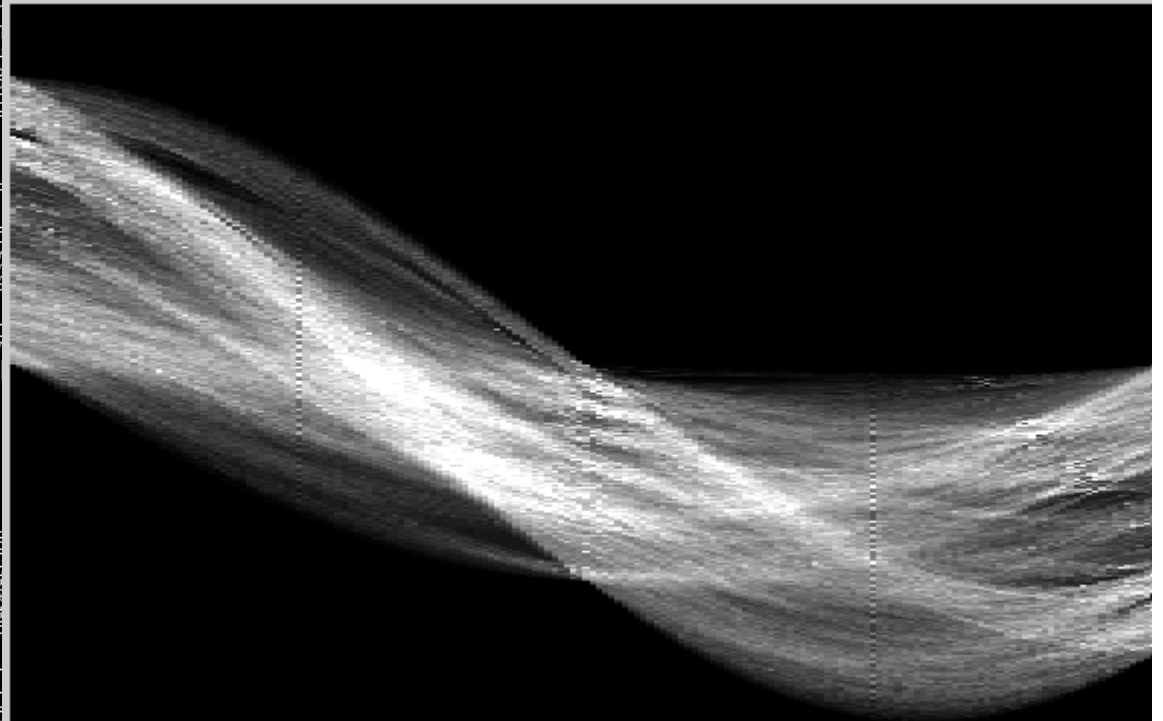
Issue: spurious peaks due to uniform noise

# 1. Image $\rightarrow$ Canny



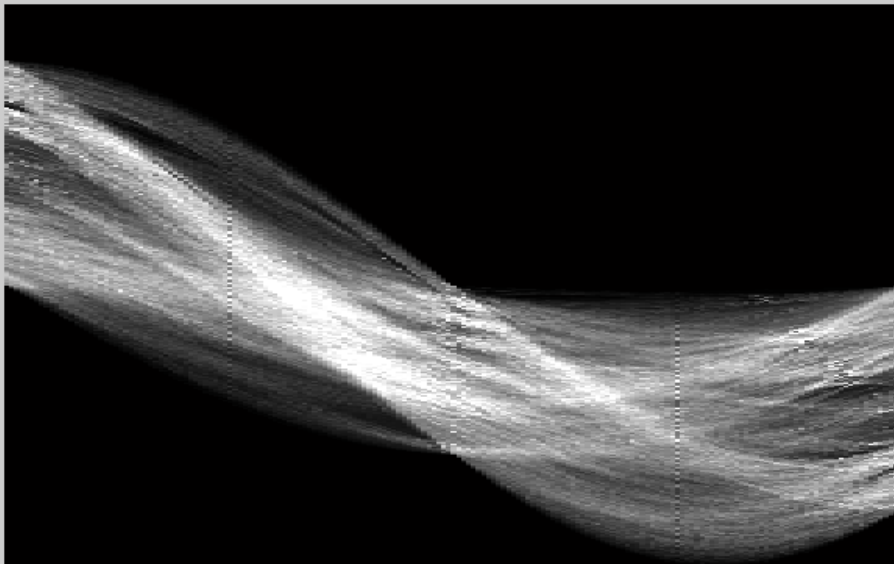


## 2. Canny $\rightarrow$ Hough votes

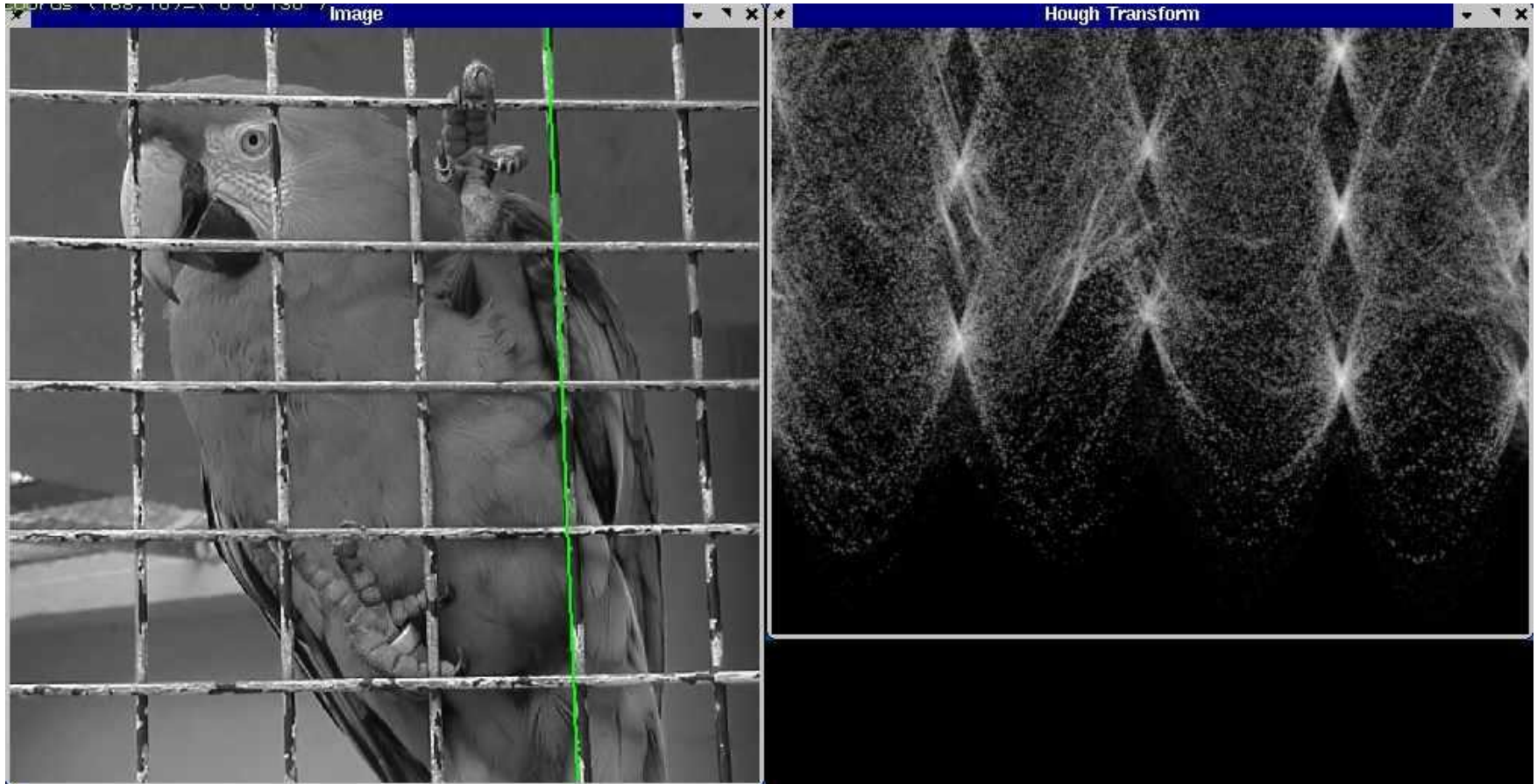


# 3. Hough votes $\rightarrow$ Edges

Find peaks and post-process



# Hough transform example



# Finding lines using Hough transform

- Using  $m, b$  parameterization
- Using  $r, \theta$  parameterization
  - Using oriented gradients
- Practical considerations
  - Bin size
  - Smoothing
  - Finding multiple lines
  - Finding line segments

# Next lecture

- RANSAC
- Connecting model fitting with feature matching