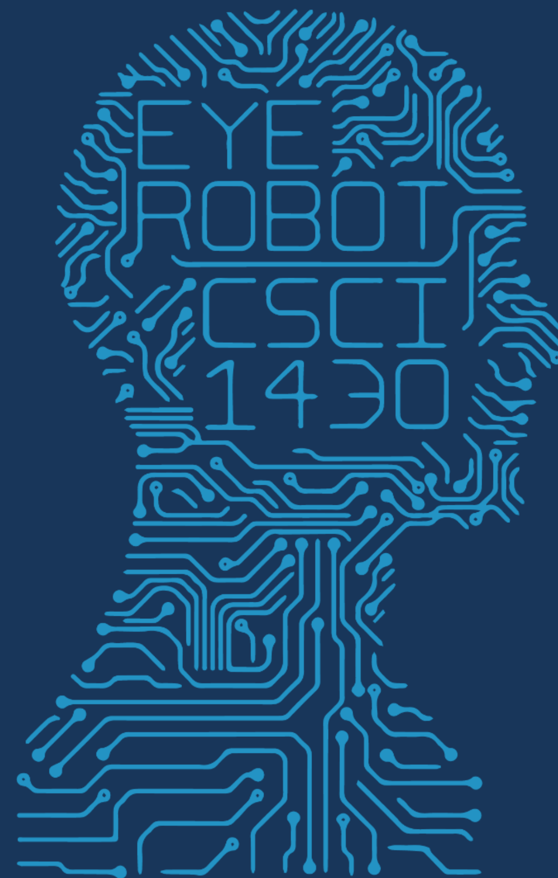
1950

Future Vision



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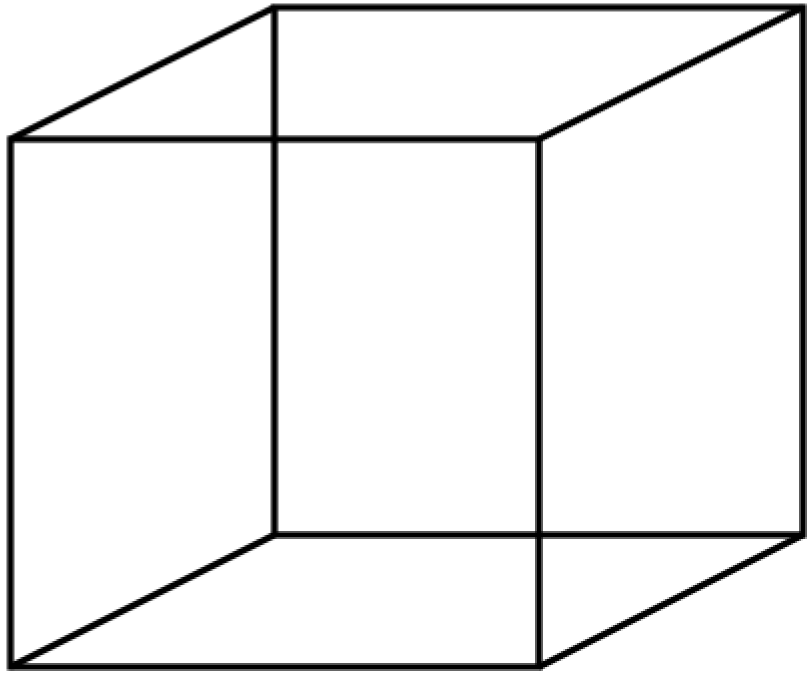
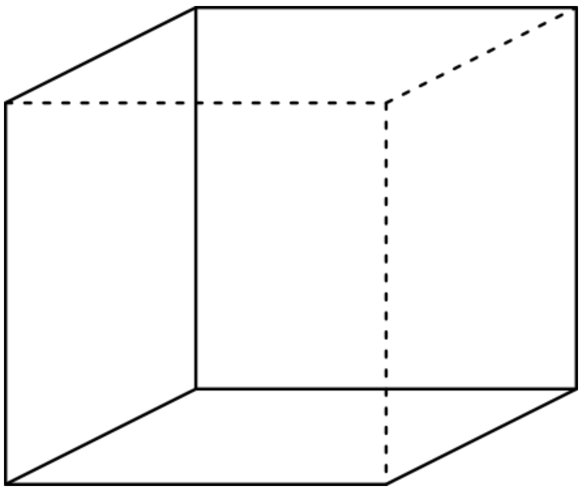
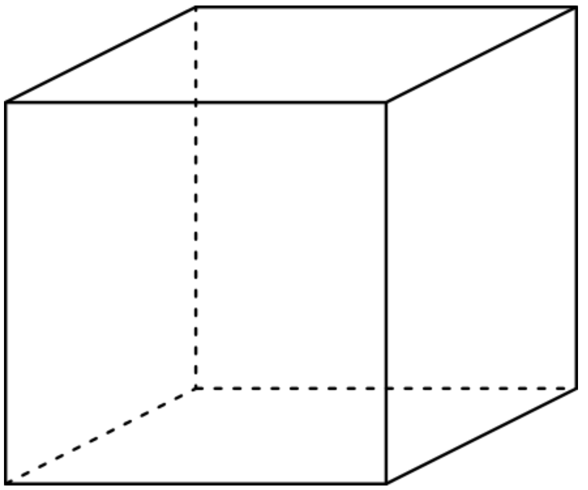
Computer Vision

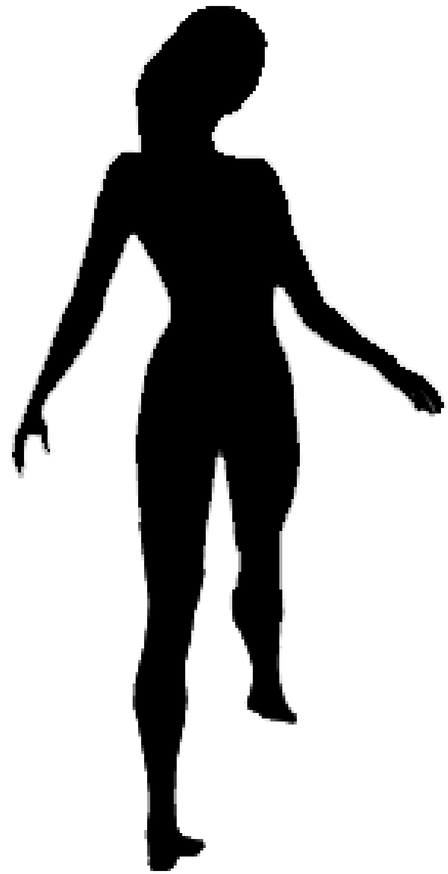
Multi-stable Perception



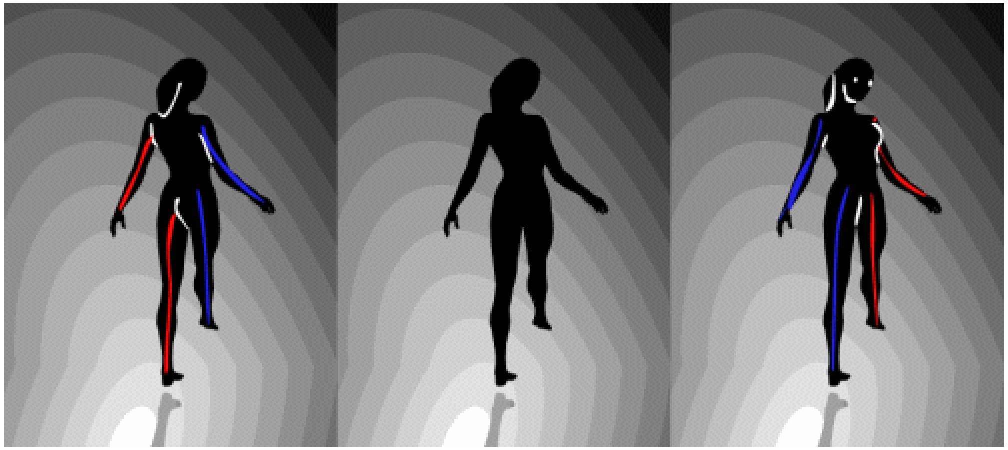
Necker Cube



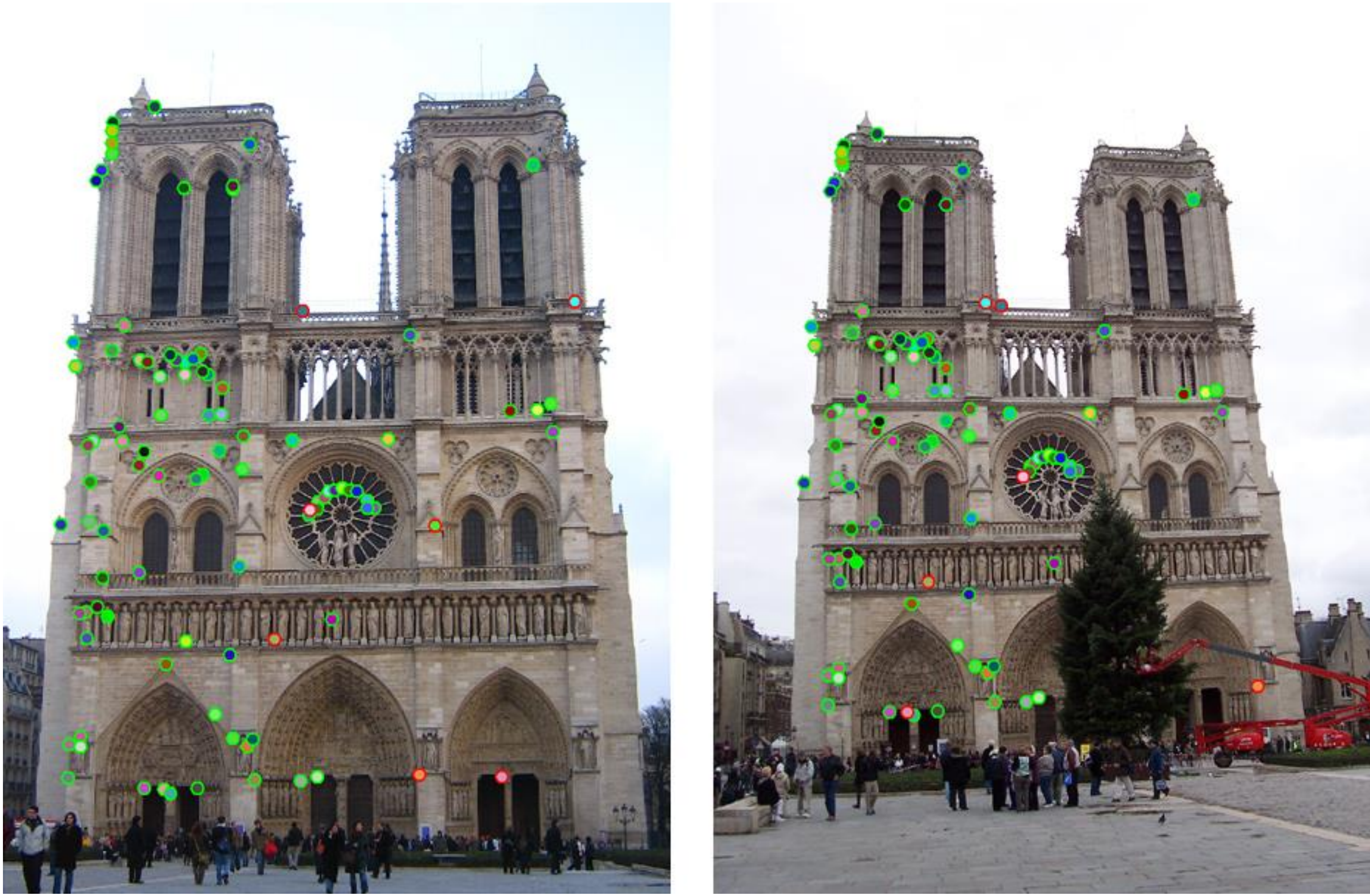




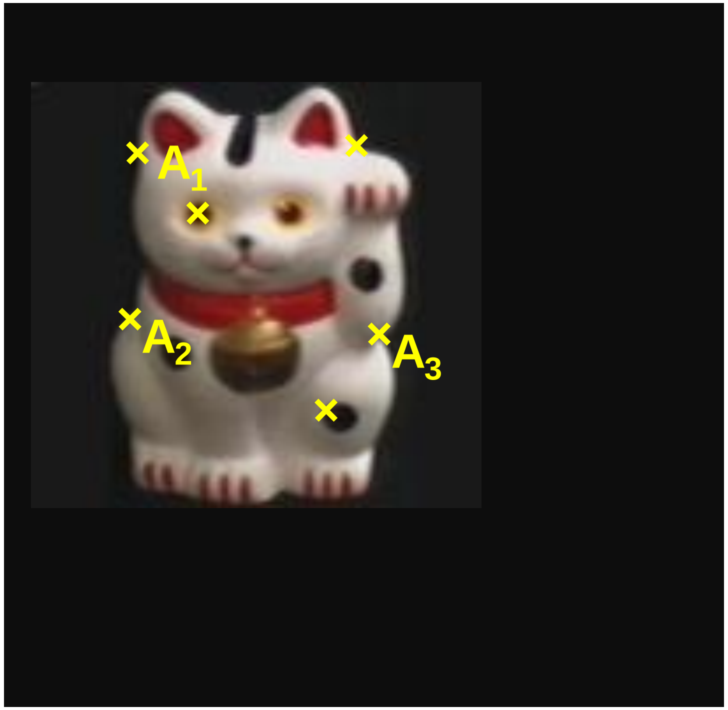
Spinning dancer illusion, Nobuyuki Kayahara



Given matches, what is the transformation?



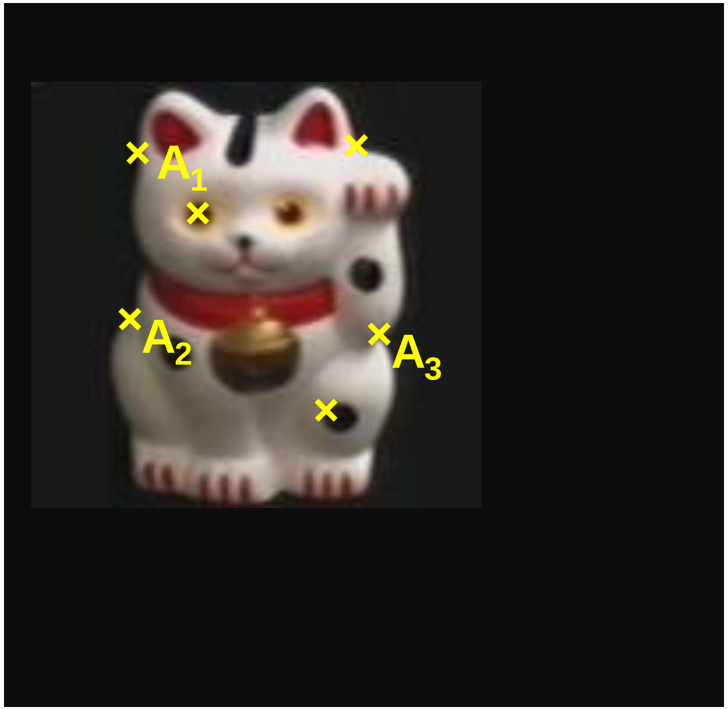
Example: discovering translation



Given matched points in {A} and {B}, estimate the translation of the object

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

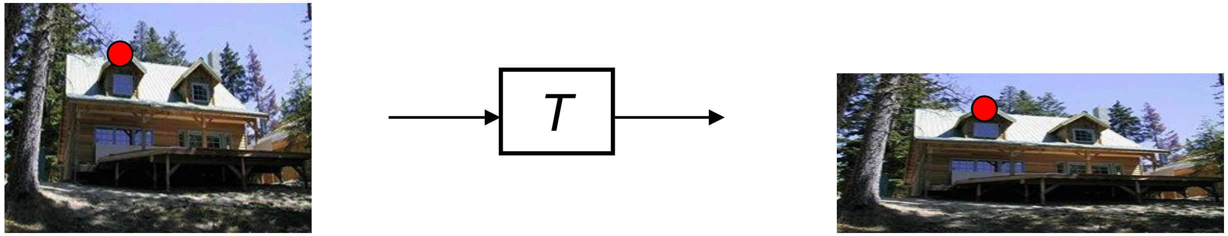
Example: discovering rot/trans/scale



Given matched points in {A} and {B}, estimate the transformation matrix

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \mathrm{T}\begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \qquad \mathrm{T} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Parametric (global) transformations



p = (x,y) **p'** = (x',y')

Transformation T is a coordinate-changing machine:

$$p' = T(p)$$

What does it mean that *T* is global?

– *T* is the same for any point p

T can be described by just a few numbers (parameters)

For linear transformations, we can represent T as a matrix

$$p' = \mathbf{T}p$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

Common transformations

Original

Transformed

Translation

Rotation

Scaling

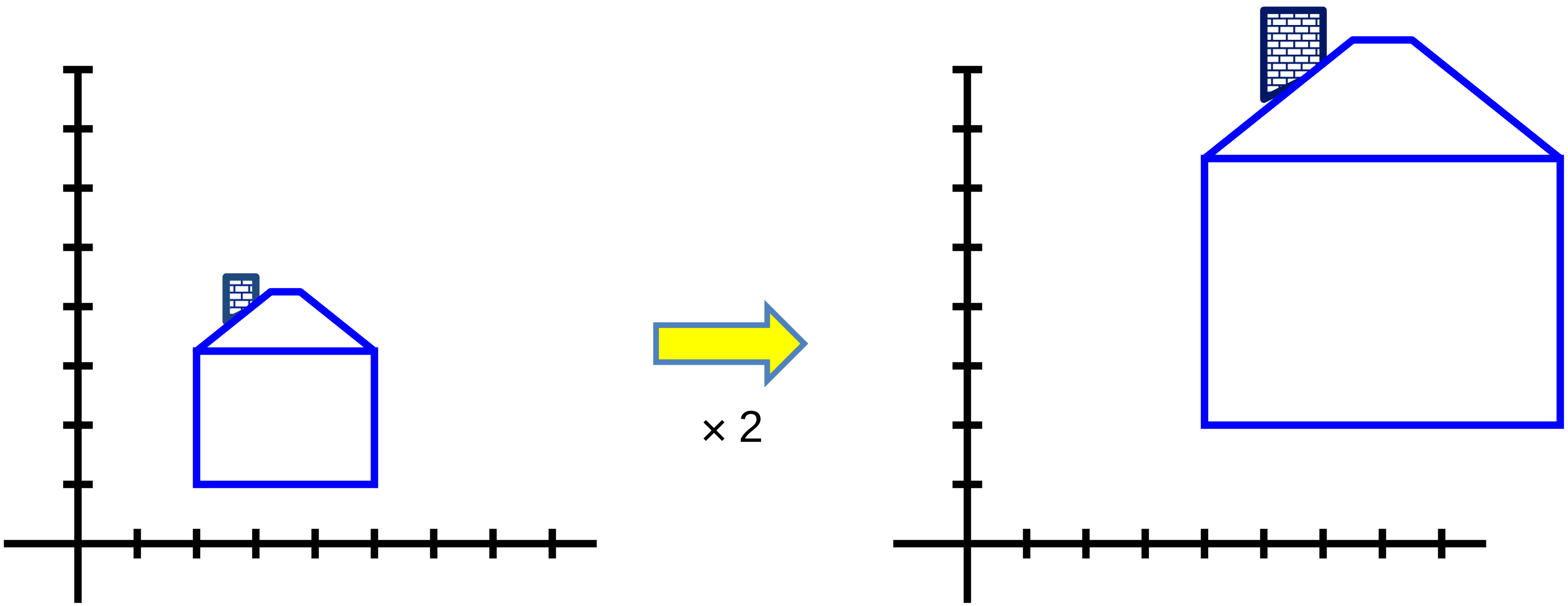
Affine

Perspective

Slide credit (next few slides):
A. Efros and/or S. Seitz

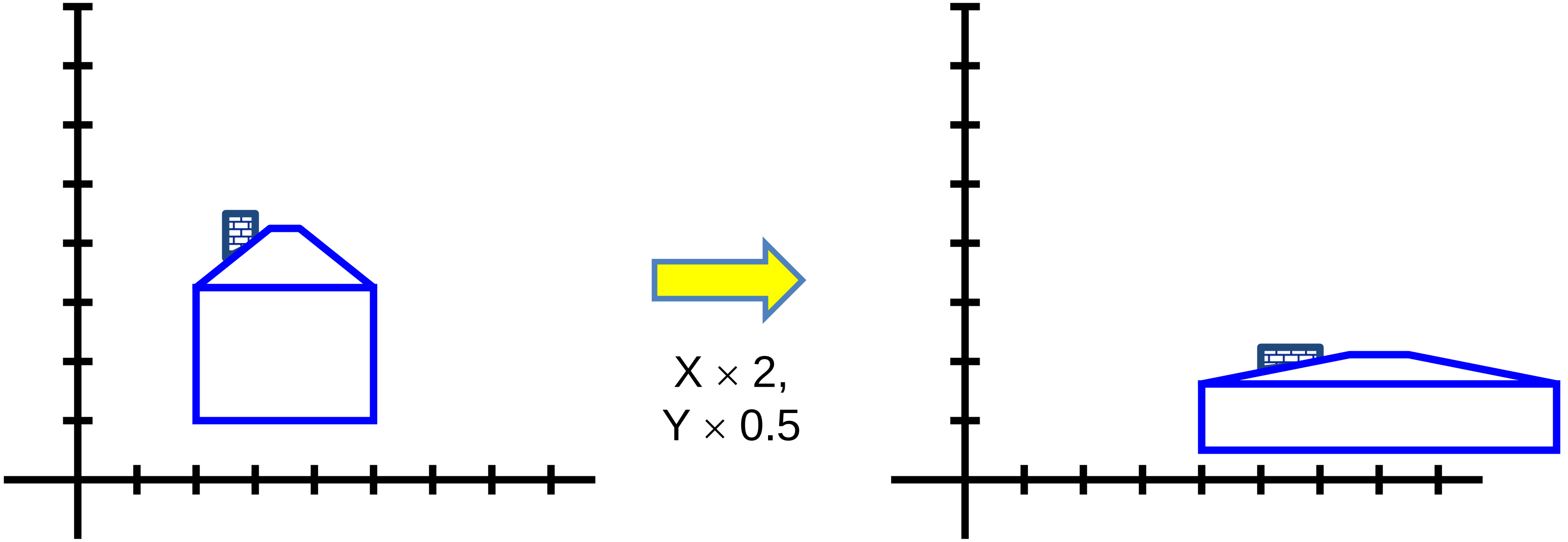
Scaling

- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:



Scaling

- *Non-uniform scaling*: different scalars per component:



Scaling

- Scaling operation:

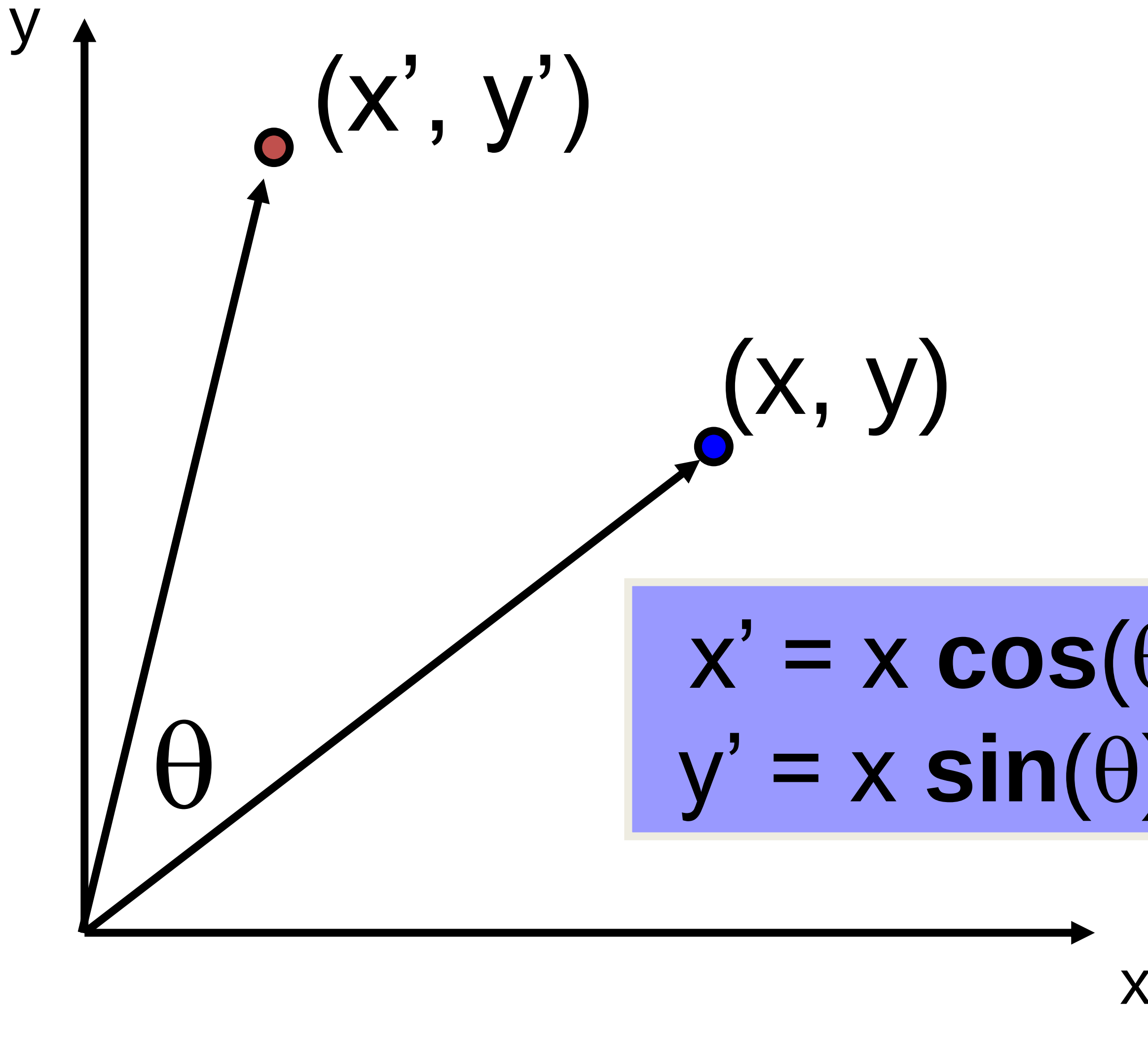
$$x' = ax$$

$$y' = by$$

- Or, in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

2-D Rotation



$$x' = x \cos(\theta) - y \sin(\theta)$$
$$y' = x \sin(\theta) + y \cos(\theta)$$

2-D Rotation

This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Even though sin(θ) and cos(θ) are nonlinear functions of θ,

- ***x' is a linear combination of x and y***
- ***y' is a linear combination of x and y***

What is the inverse transformation?

- Rotation by –θ
- For rotation matrices $\mathbf{R}^{-1} = \mathbf{R}^T$

Basic 2D transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \alpha_x \\ \alpha_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine

Affine is any combination of translation, scale, rotation, and shear

Affine Transformations

Affine transformations are combinations of

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

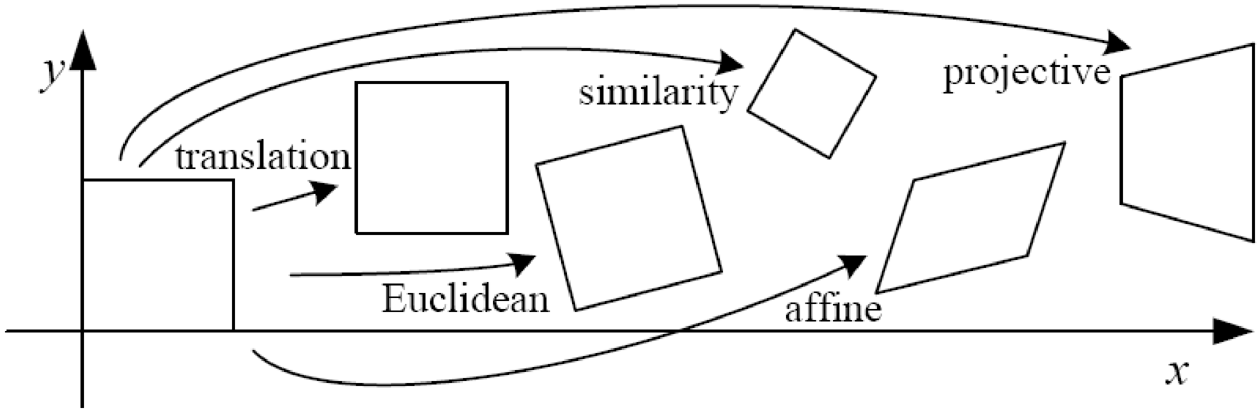
or

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Properties of affine transformations:

- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

2D image transformations (reference table)



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation $+ \cdots$	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths $+ \cdots$	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles $+ \cdots$	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism $+ \cdots$	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

‘Homography’

Szeliski 2.1

Projective Transformations

Projective transformations are combos of

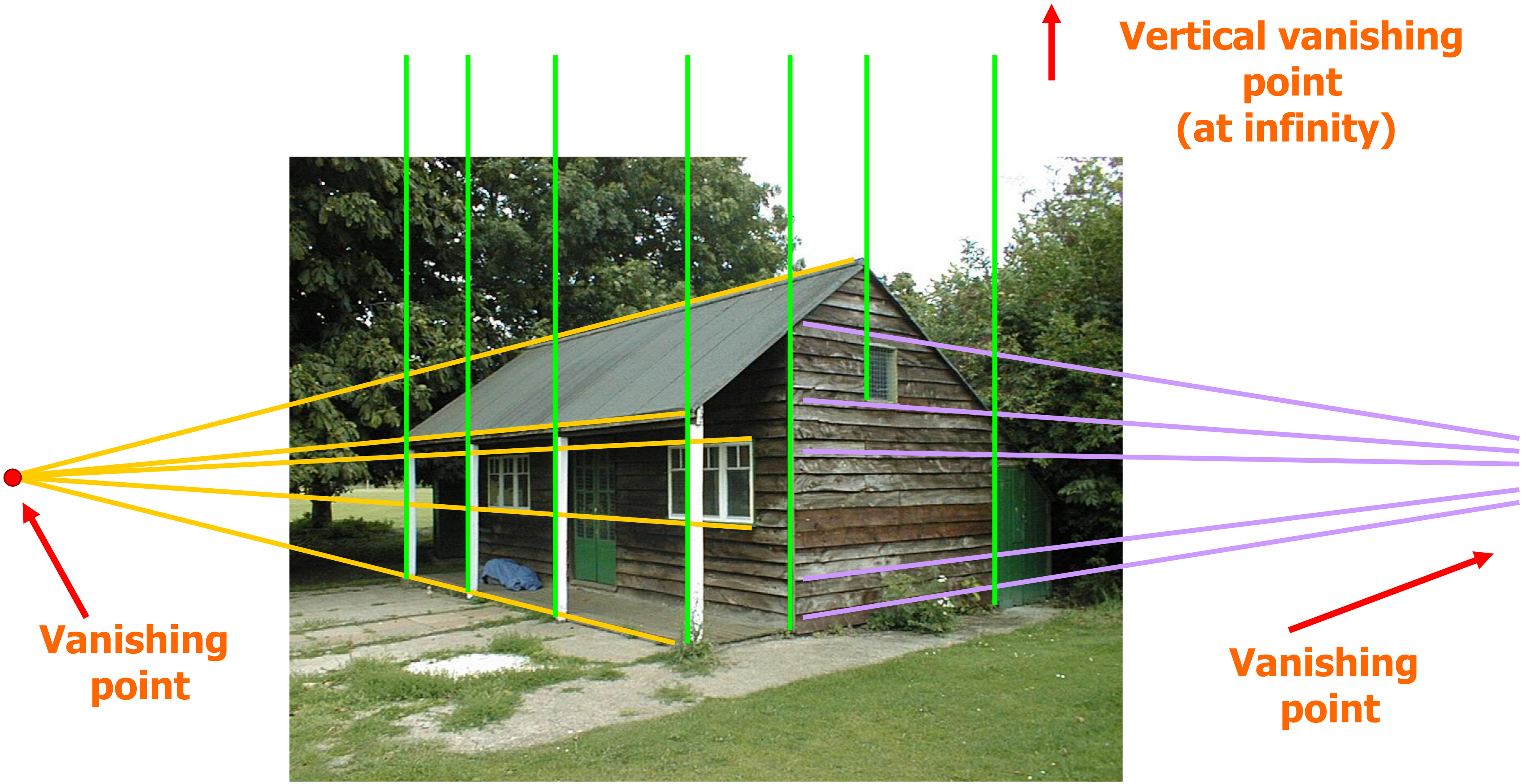
- Affine transformations, and
- Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of projective transformations:

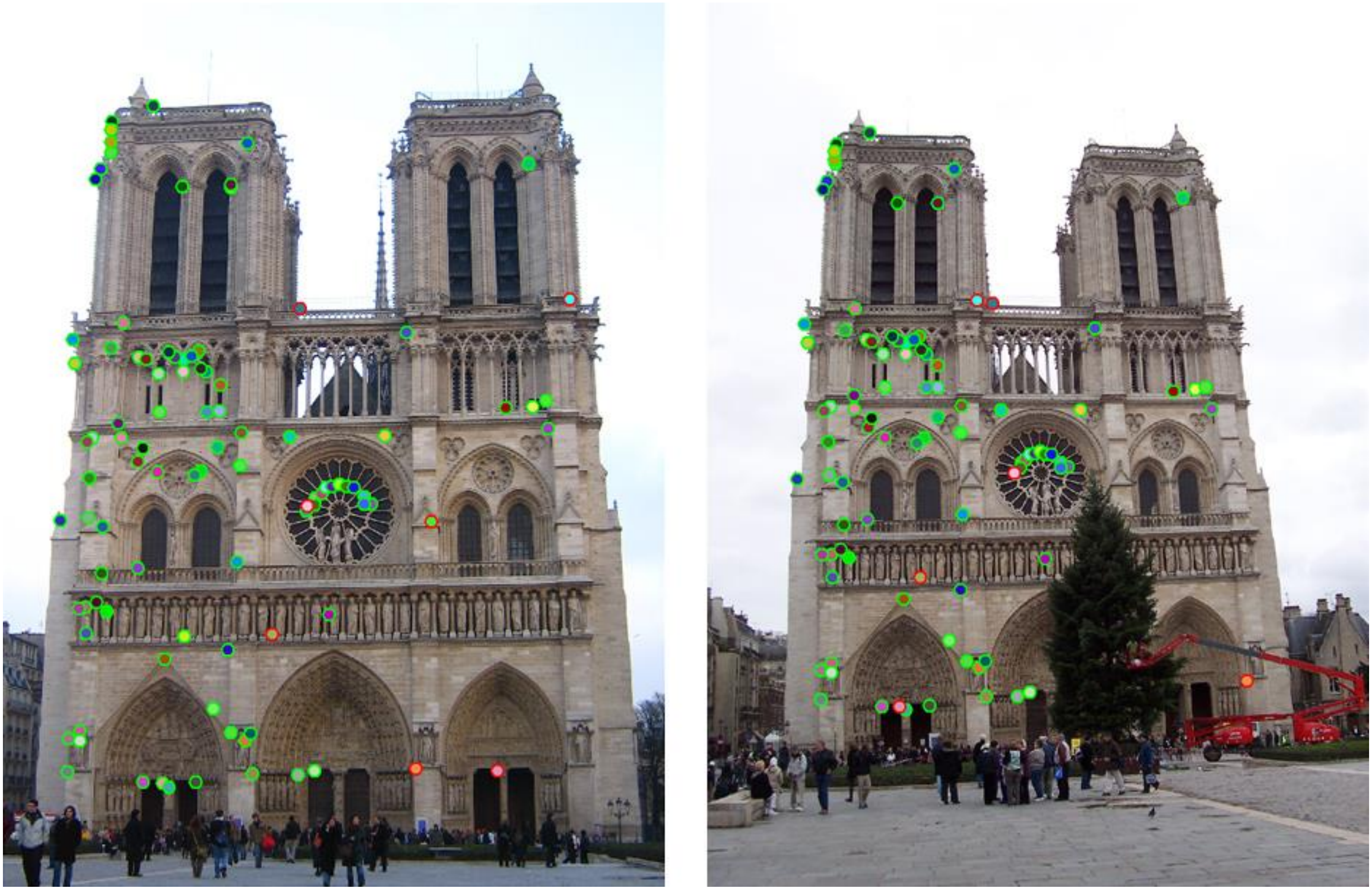
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis
- Projective matrix is defined up to a scale (8 DOF)

Example: vanishing points and lines



Slide from Efros, Photo from Criminisi

Given matches, what is the transformation?



Fitting and Alignment

Fitting:

Find the parameters of a model that best fit the data.

Alignment:

Find the parameters of the transformation that best aligns matched points.

Fitting and Alignment

- Challenges
  - Design a suitable **goodness of fit** measure
    - Similarity should reflect application goals
    - Encode robustness to outliers and noise
  - Design an **optimization** method
    - Avoid local optima
    - Find best parameters quickly
  - Typically want to solve for a global transformation that accounts for **the most** true correspondences
    - Noise (typically 1-3 pixels)
    - Outliers (often 50%)
    - Many-to-one matches or multiple objects

Fitting and Alignment: Methods

- Global optimization / search for parameters
  - Least squares fit
  - Robust least squares
  - Iterative closest point (ICP)

- Hypothesize and test
  - Generalized Hough transform
  - RANSAC

Fitting and Alignment: Methods

- Global optimization / search for parameters
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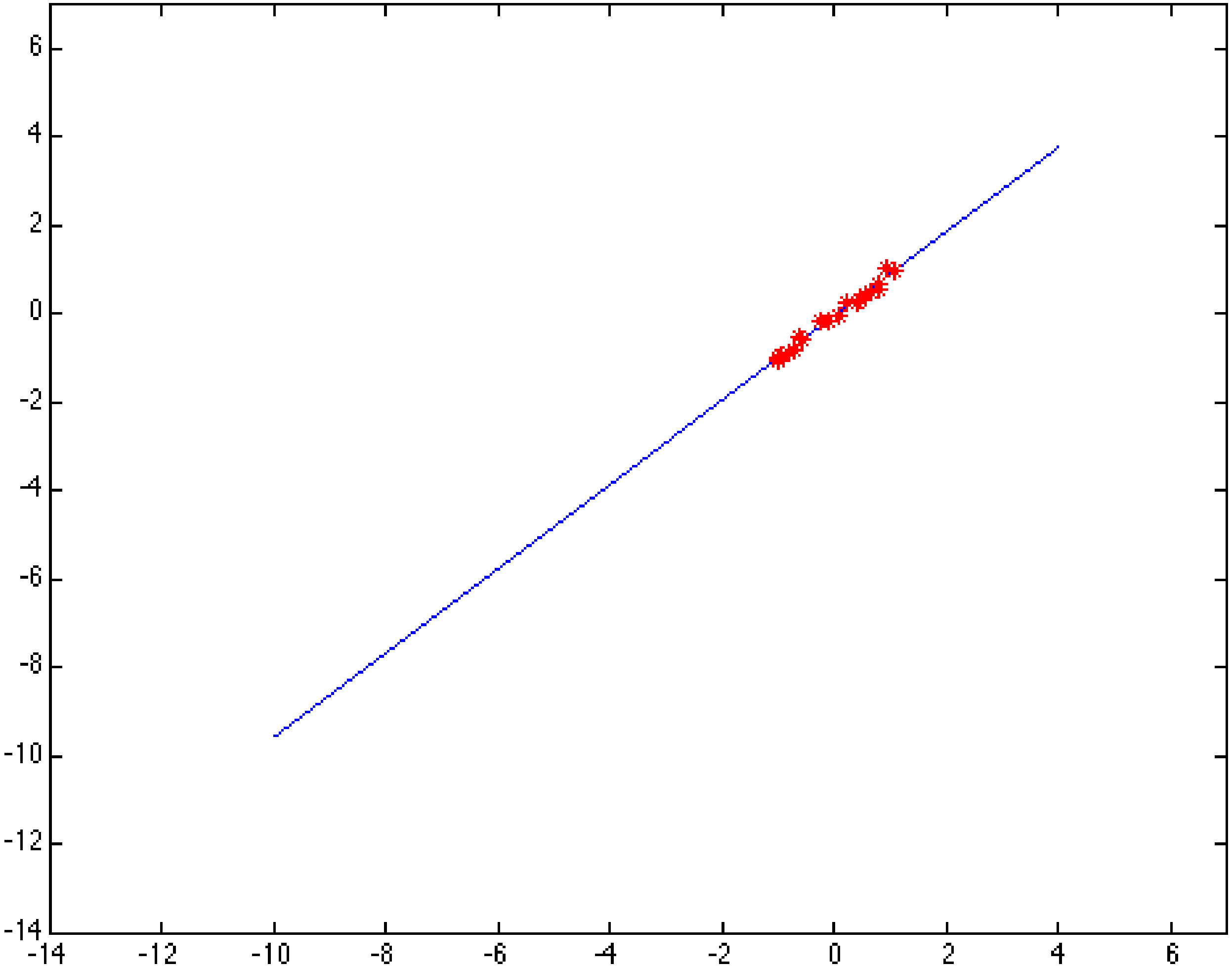
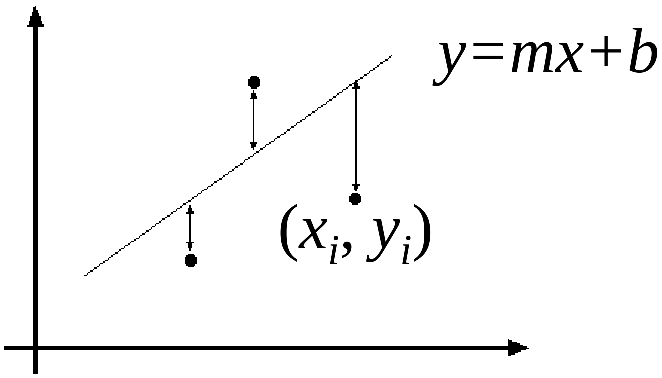
Simple example: Fitting a line



Least squares line fitting

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m\, x_i + b$
- Find (m, b) to minimize



$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$

$$E = \sum_{i=1}^{n} \left(\begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - y_i \right)^2 = \left\| \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right\|^2 = \left\| \mathbf{Ap} - \mathbf{y} \right\|^2$$

$$= \mathbf{y}^T \mathbf{y} - 2(\mathbf{Ap})^T \mathbf{y} + (\mathbf{Ap})^T (\mathbf{Ap})$$

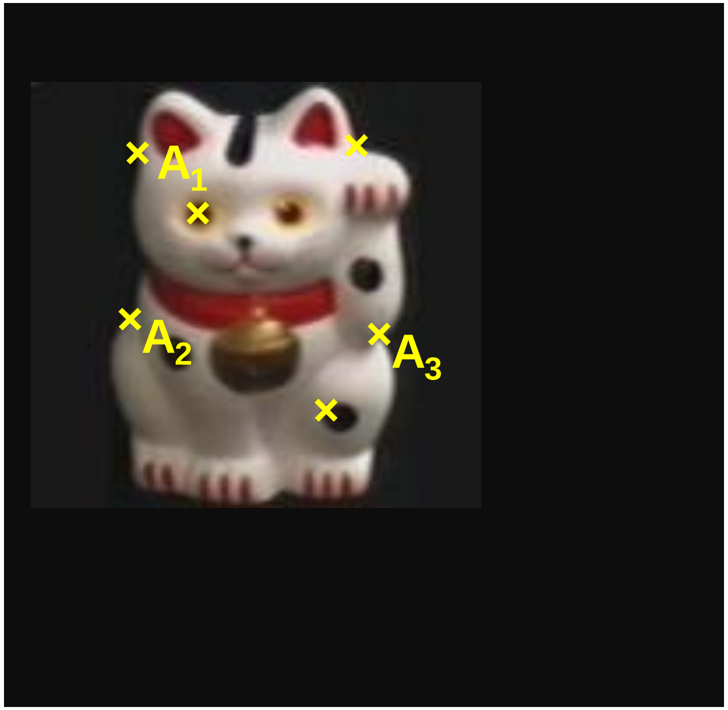
$$\frac{dE}{dp} = 2\mathbf{A}^T \mathbf{Ap} - 2\mathbf{A}^T \mathbf{y} = 0$$

Matlab: `p = A \ y;`

$$\mathbf{A}^T \mathbf{Ap} = \mathbf{A}^T \mathbf{y} \Rightarrow \mathbf{p} = \left(\mathbf{A}^T \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{y}$$ (Closed form solution)

Modified from S. Lazebnik

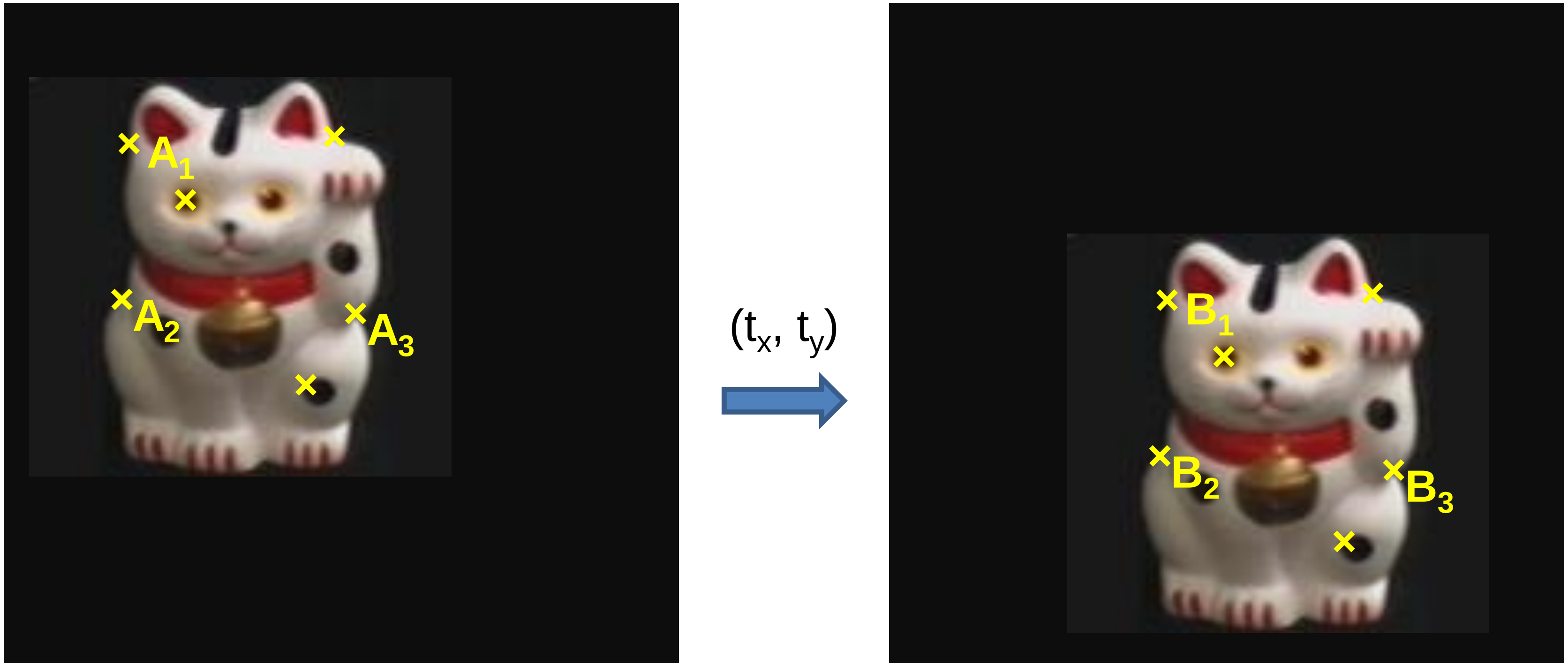
Example: solving for translation



Given matched points in {A} and {B}, estimate the translation of the object

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



Least squares solution

1. Write down objective function
2. Derived solution
   a) Compute derivative
   b) Compute solution
3. Computational solution
   a) Write in form Ax=p
   b) Solve using closed-form solution

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x_1^B - x_1^A \\ y_1^B - y_1^A \\ \vdots \\ x_n^B - x_n^A \\ y_n^B - y_n^A \end{bmatrix}$$

Least squares (global) optimization

Good

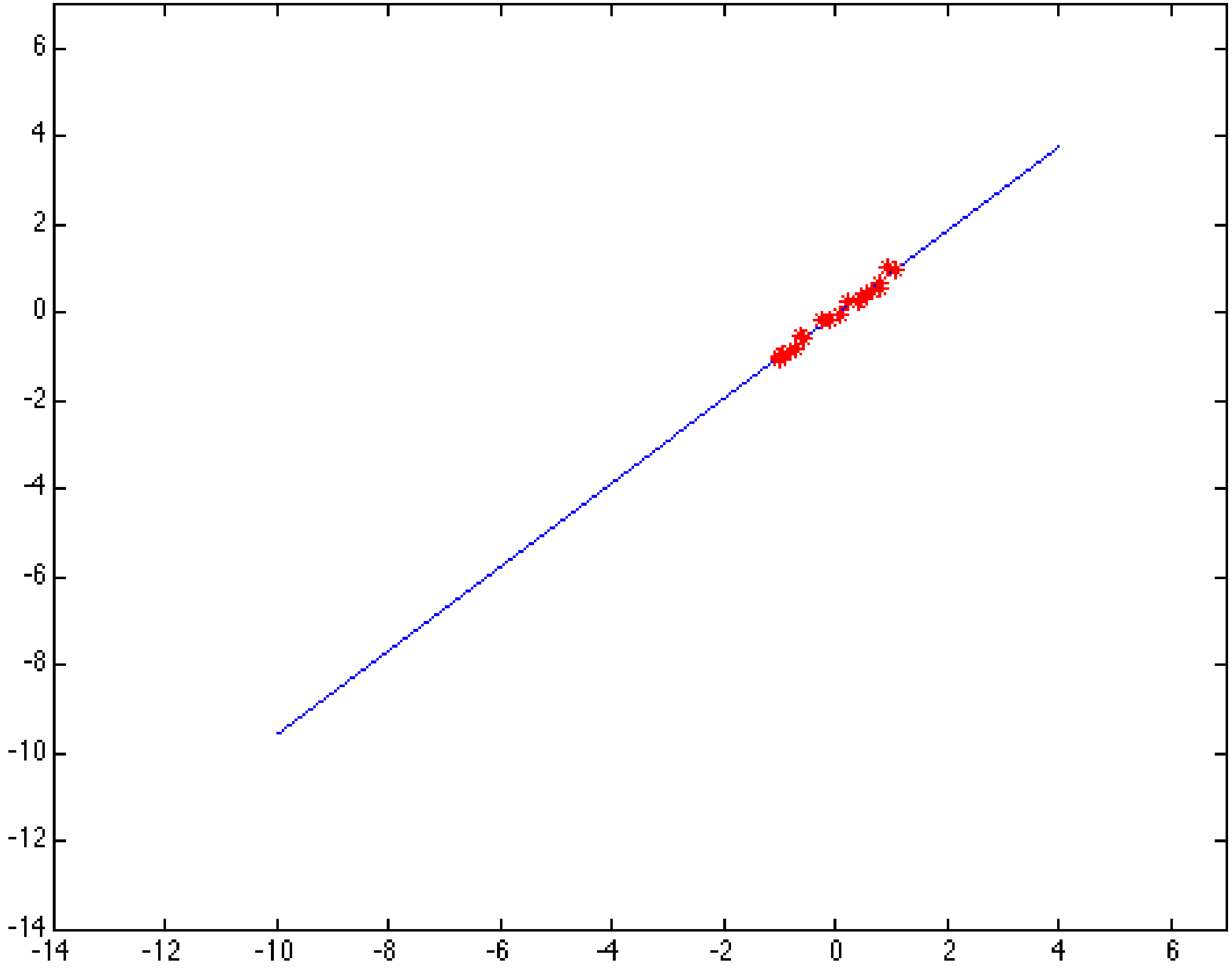
- Clearly specified objective
- Optimization is easy

Bad

- Sensitive to outliers
  - Bad matches, extra points
- Doesn’t allow you to get multiple good fits
  - Detecting multiple objects, lines, etc.

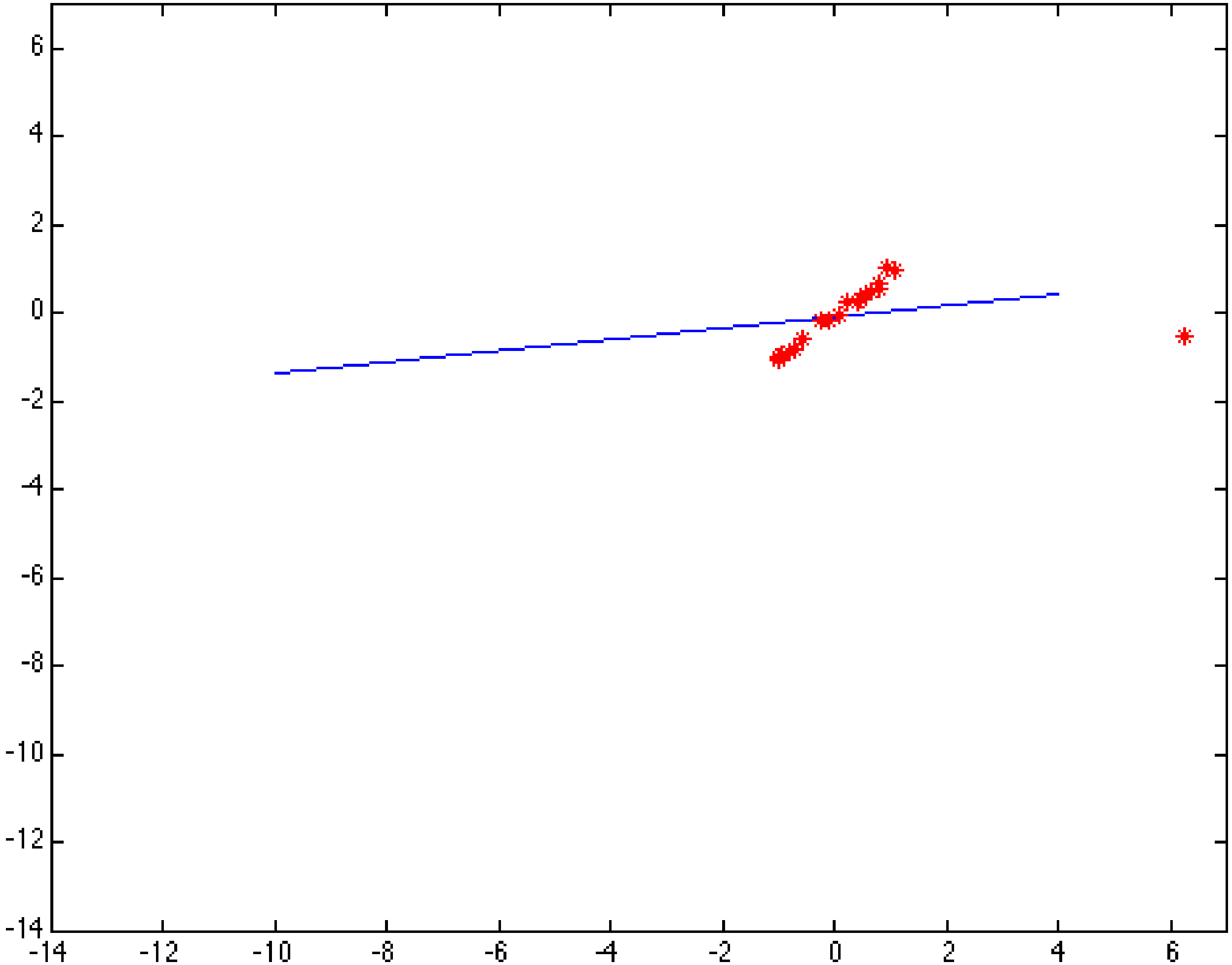
Least squares: Robustness to noise

- Least squares fit to the red points:



Least squares: Robustness to noise

- Least squares fit with an outlier:

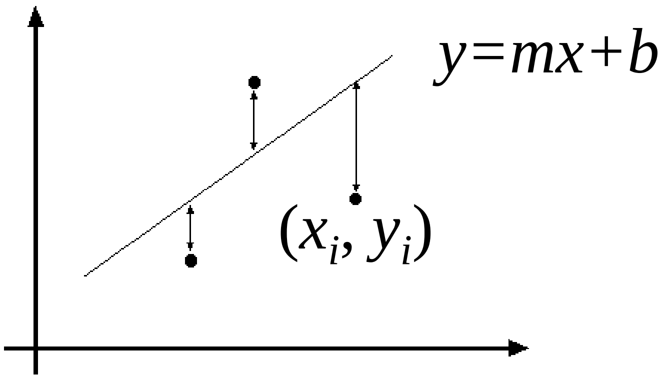


Problem: squared error heavily penalizes outliers

Least squares line fitting

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m\,x_i + b$
- Find (m, b) to minimize

$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$



Matlab: `p = A \ y;`

(Closed form solution)

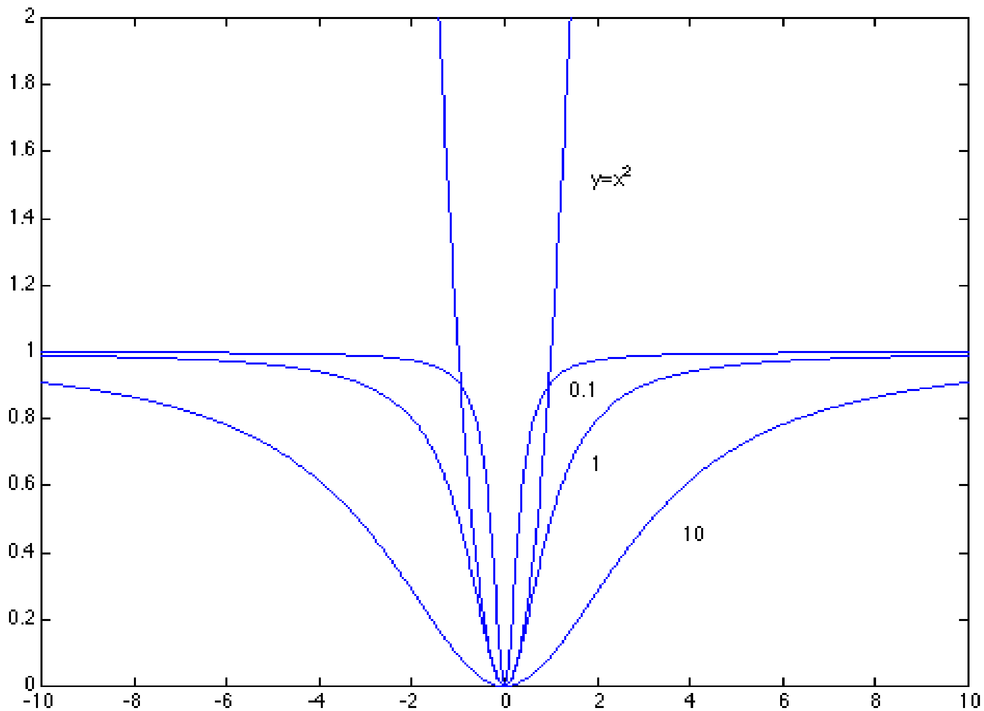
Robust least squares (to deal with outliers)

General approach:

minimize $$\sum_i \rho\left(u_i\left(x_i,\theta\right);\sigma\right)$$ $$u^2 = \sum_{i=1}^{n}(y_i - mx_i - b)^2$$

$u_i\,(x_i,\,\theta)$ – residual of i^th point w.r.t. model parameters θ

ρ – robust function with scale parameter σ



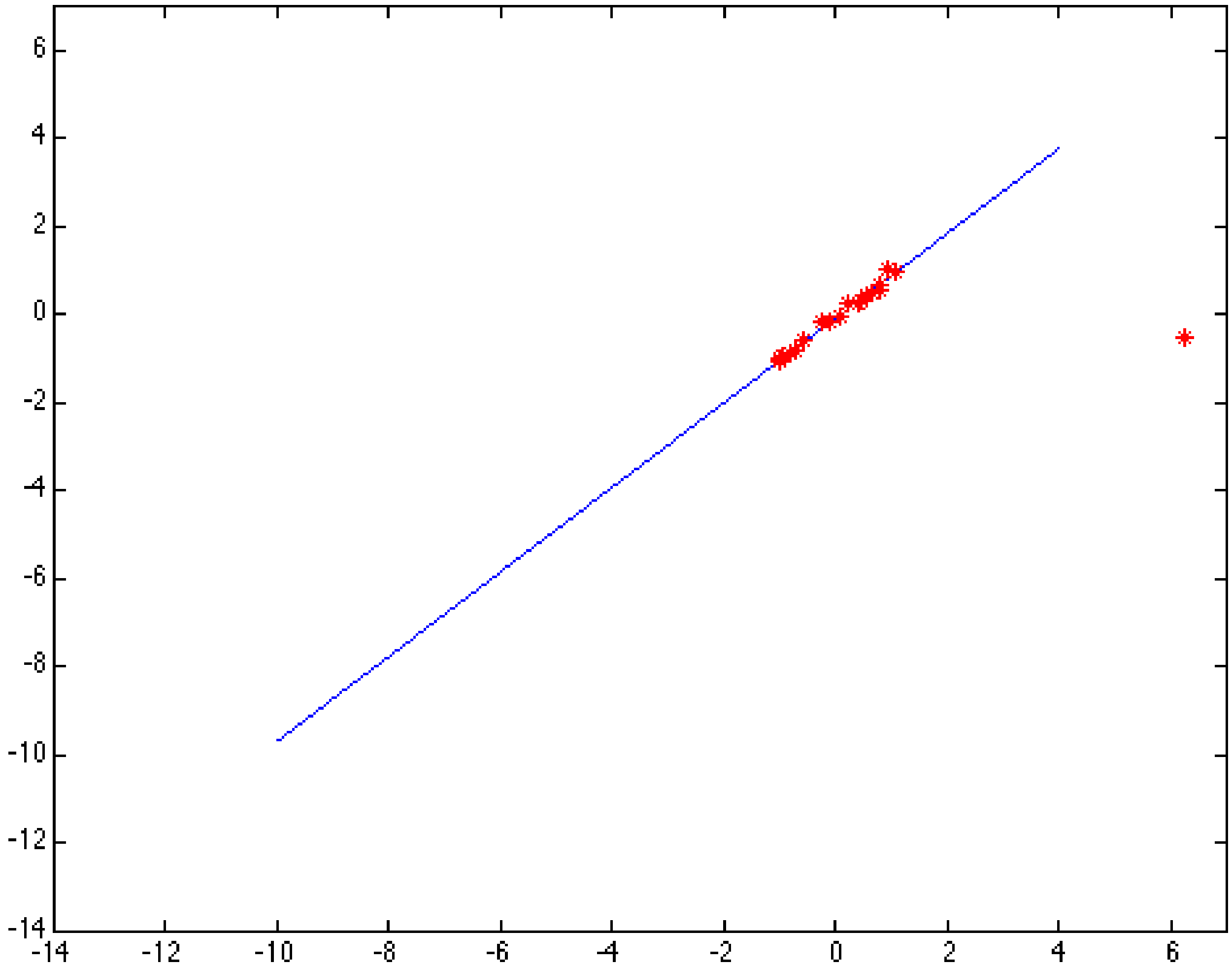
The robust function ρ

- Favors a configuration with small residuals
- Constant penalty for large residuals

$$\boxed{\rho(u;\sigma) = \frac{u^2}{\sigma^2 + u^2}}$$

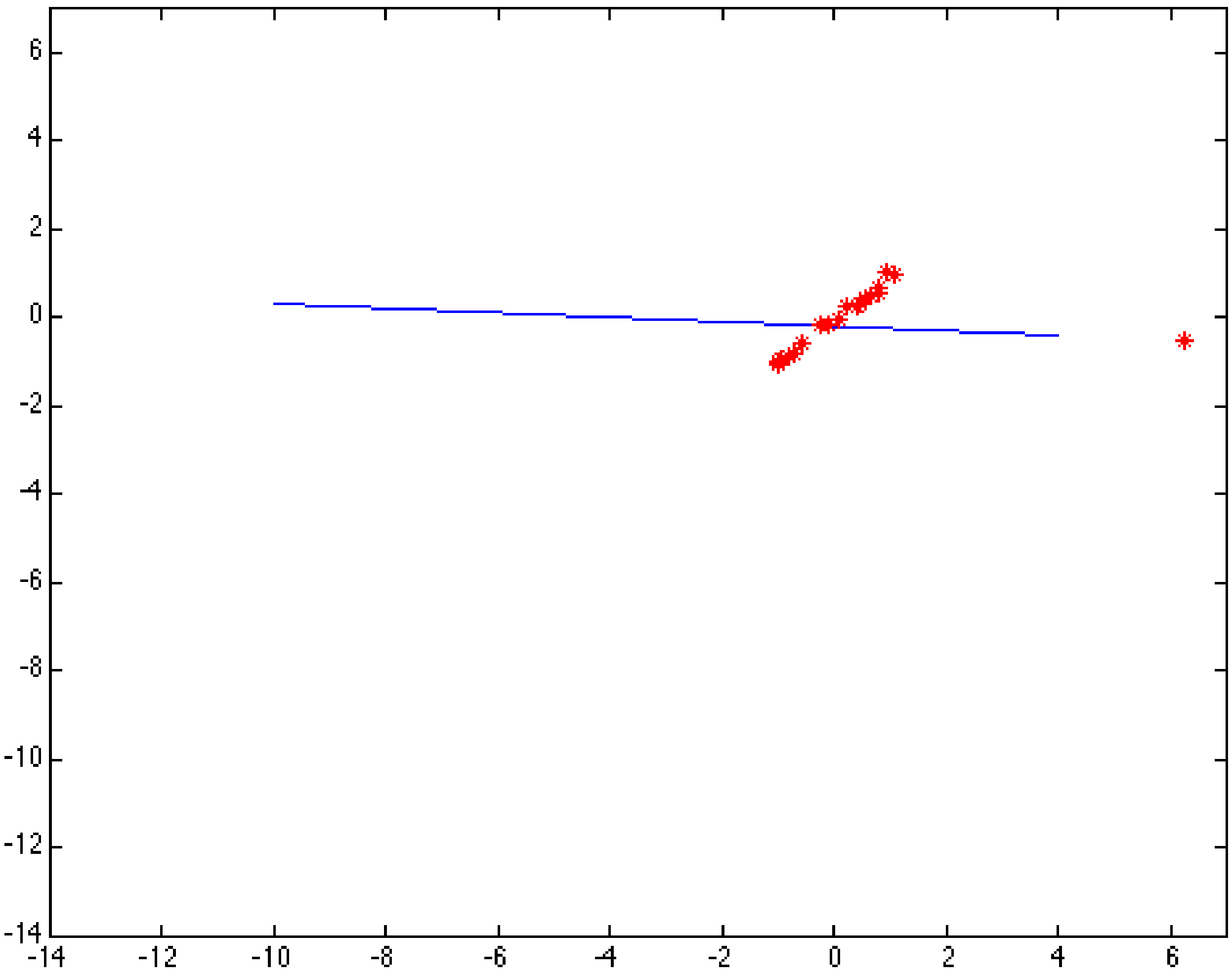
Slide from S. Savarese

Choosing the scale: Just right



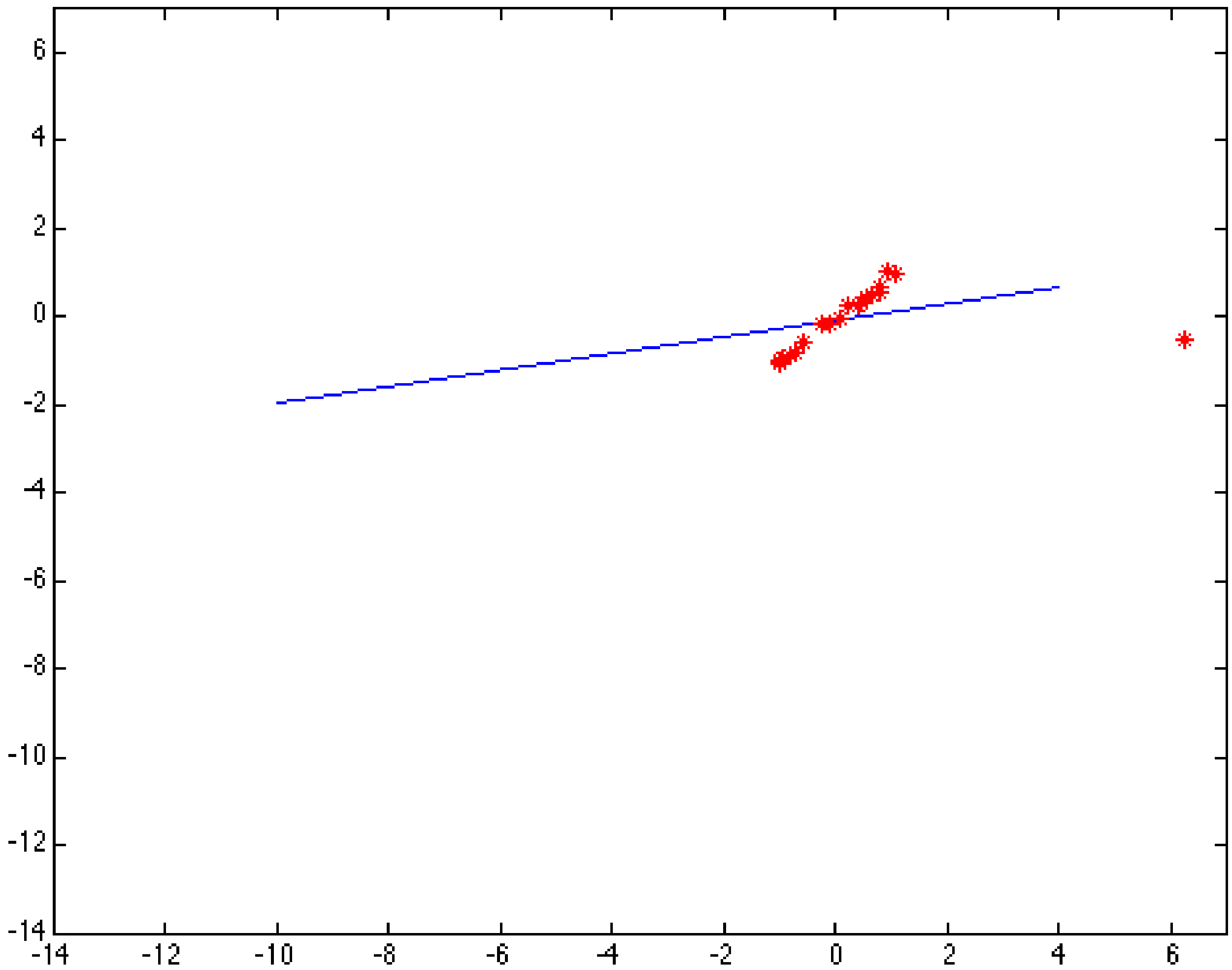
The effect of the outlier is minimized

Choosing the scale: Too small



The error value is almost the same for every point and the fit is very poor

Choosing the scale: Too large



Behaves much the same as least squares

Robust estimation: Details

- Robust fitting is a nonlinear optimization problem that must be solved iteratively
- Scale of robust function should be chosen adaptively based on median residual
- Least squares solution can be used for initialization

Other ways to search for parameters for when no closed form solution exists

Line search

1. For each parameter, step through values and choose value that gives best fit
2. Repeat (1) until no parameter changes

Grid search

1. Propose several sets of parameters, evenly sampled in the joint set
2. Choose best (or top few) and sample joint parameters around the current best; repeat

Gradient descent

1. Provide initial position (e.g., random)
2. Locally search for better parameters by following gradient

Hypothesize and test

1. Propose parameters
   – Try all possible
   – Each point votes for all consistent parameters
   – Repeatedly sample enough points to solve for parameters

2. Score the given parameters
   – Number of consistent points, possibly weighted by distance

3. Choose from among the set of parameters
   – Global or local maximum of scores

4. Possibly refine parameters using inliers

Fitting and Alignment: Methods

- Global optimization / search for parameters
  - Least squares fit
  - Robust least squares
  - Iterative closest point (ICP)
- Hypothesize and test
  - Generalized Hough transform
  - RANSAC

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in ‘81.

RANSAC

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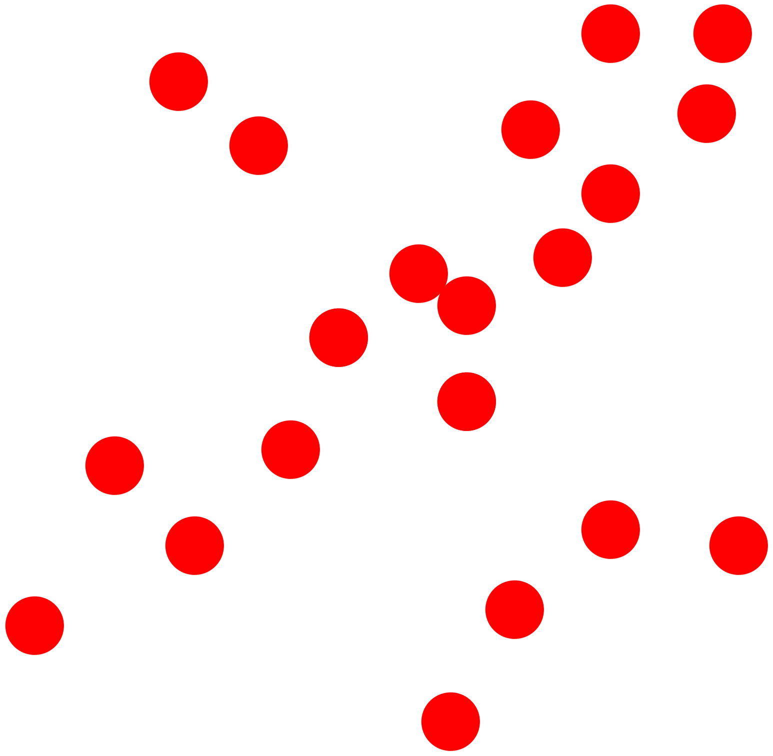
Fischler & Bolles in ‘81.



RANSAC

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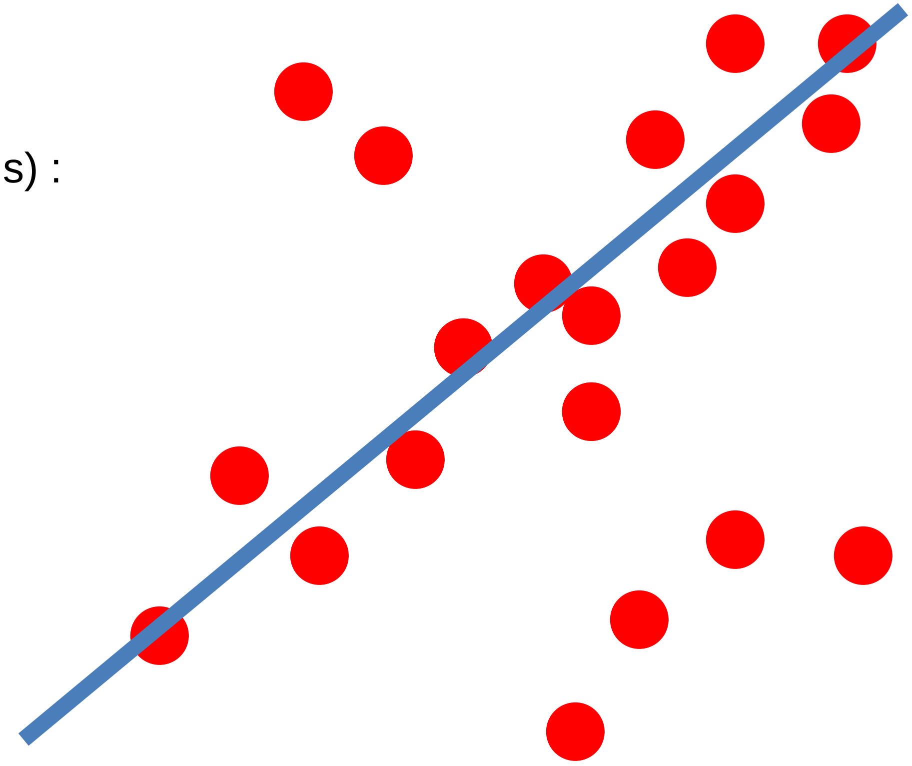

This data is noisy, but we expect a good fit to a known model.

RANSAC

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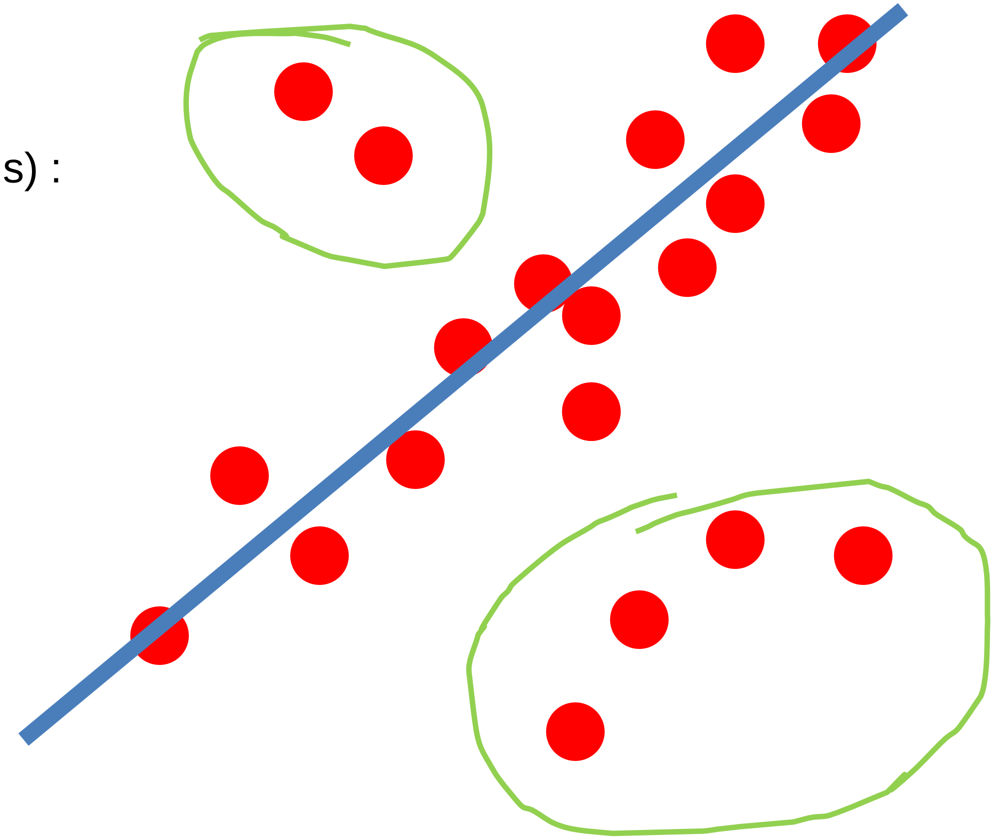


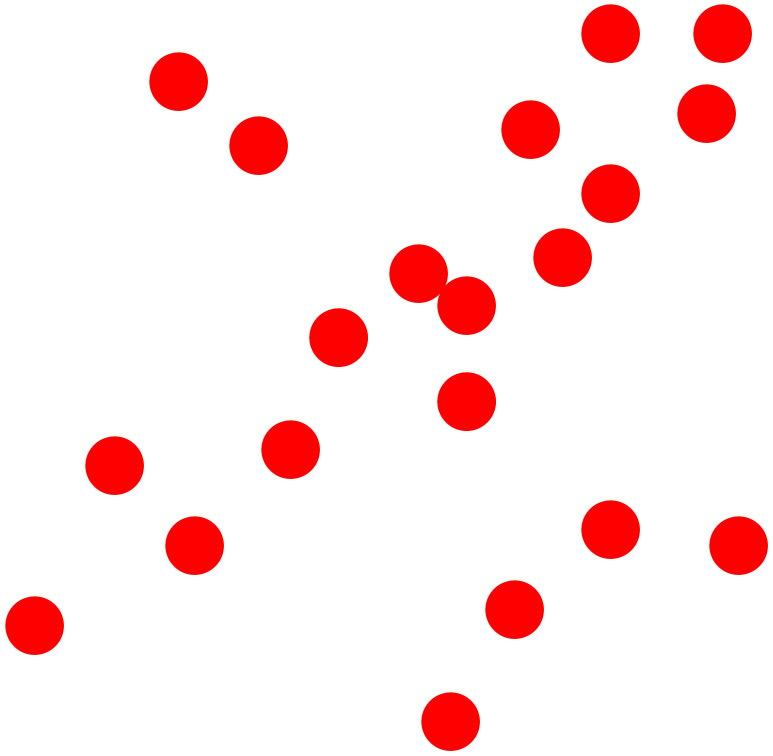
This data is noisy, but we expect a good fit to a known model.

Here, we expect to see a line, but least-squares fitting will produce the wrong result due to strong outlier presence.

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in ‘81.



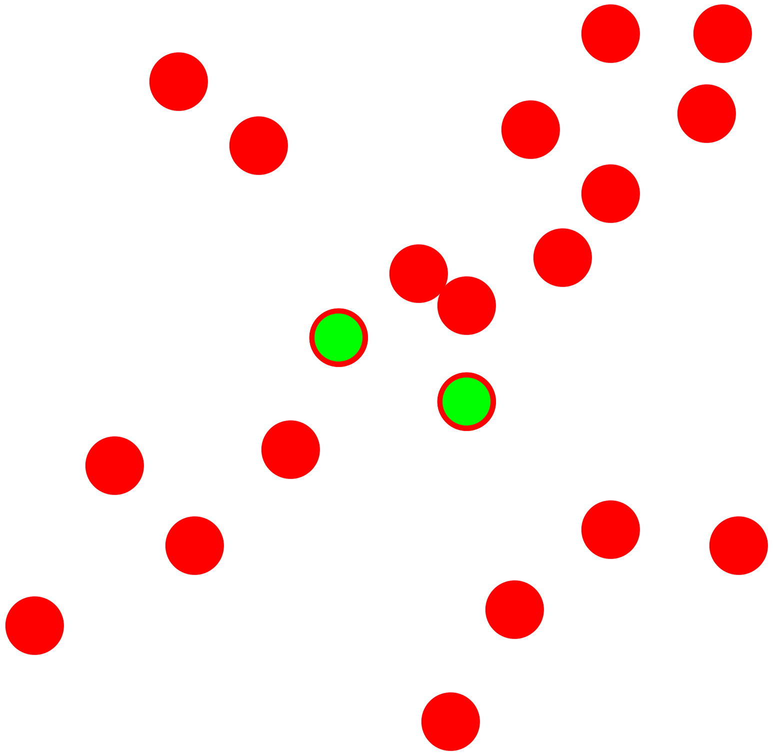
Algorithm:

1. **Sample** (randomly) the number of points *s* required to fit the model
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

RANSAC

Line fitting example



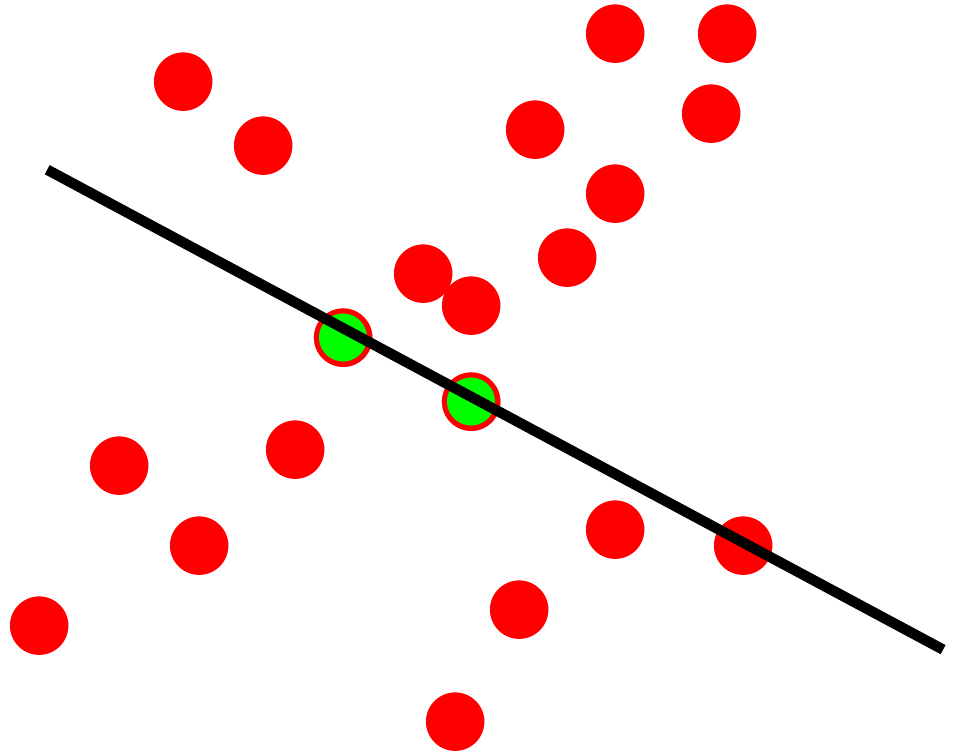
Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (s=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

RANSAC



Line fitting example

Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (*s*=2)
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3. **Score** by the fraction of inliers within a preset threshold of the model

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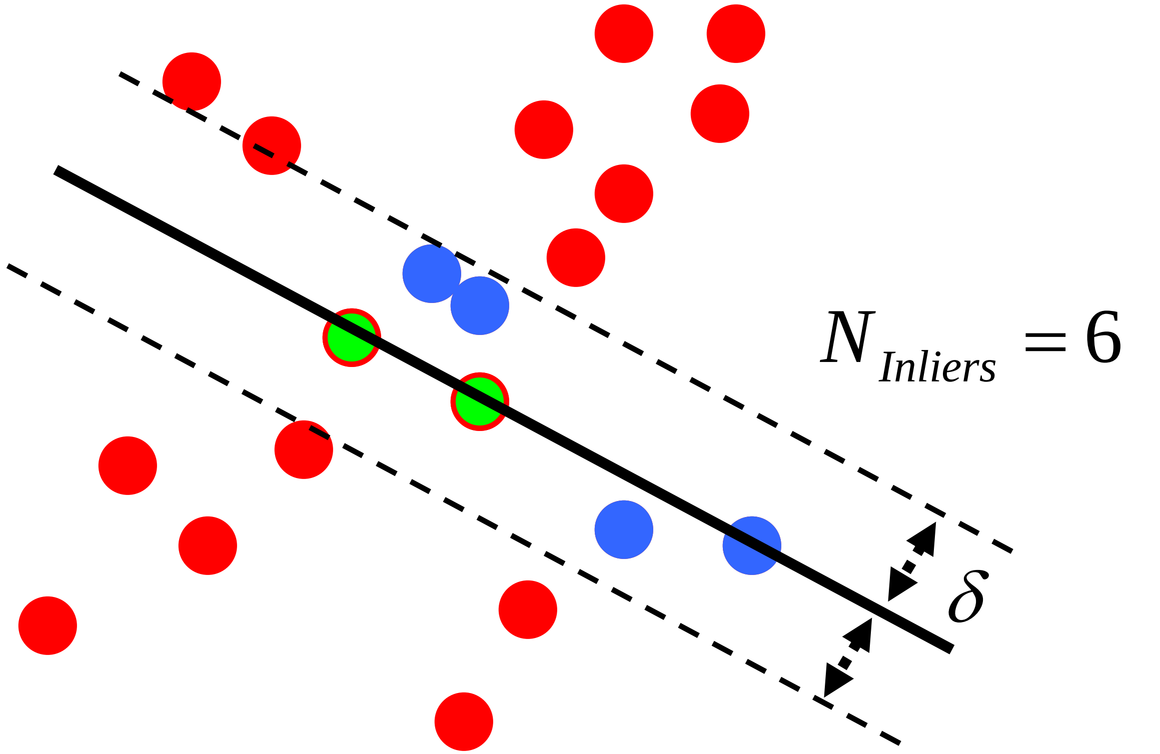
RANSAC

Line fitting example



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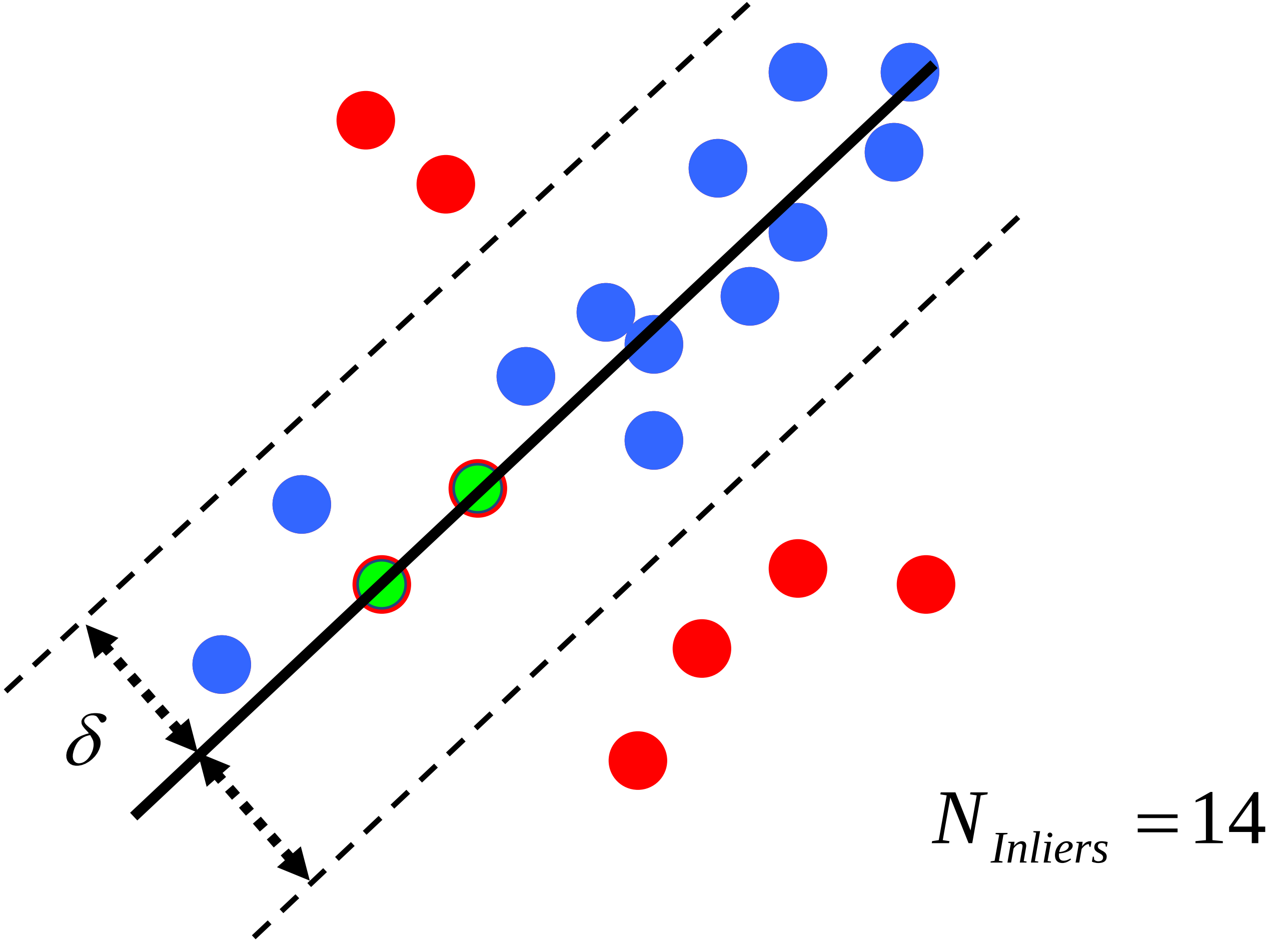
RANSAC



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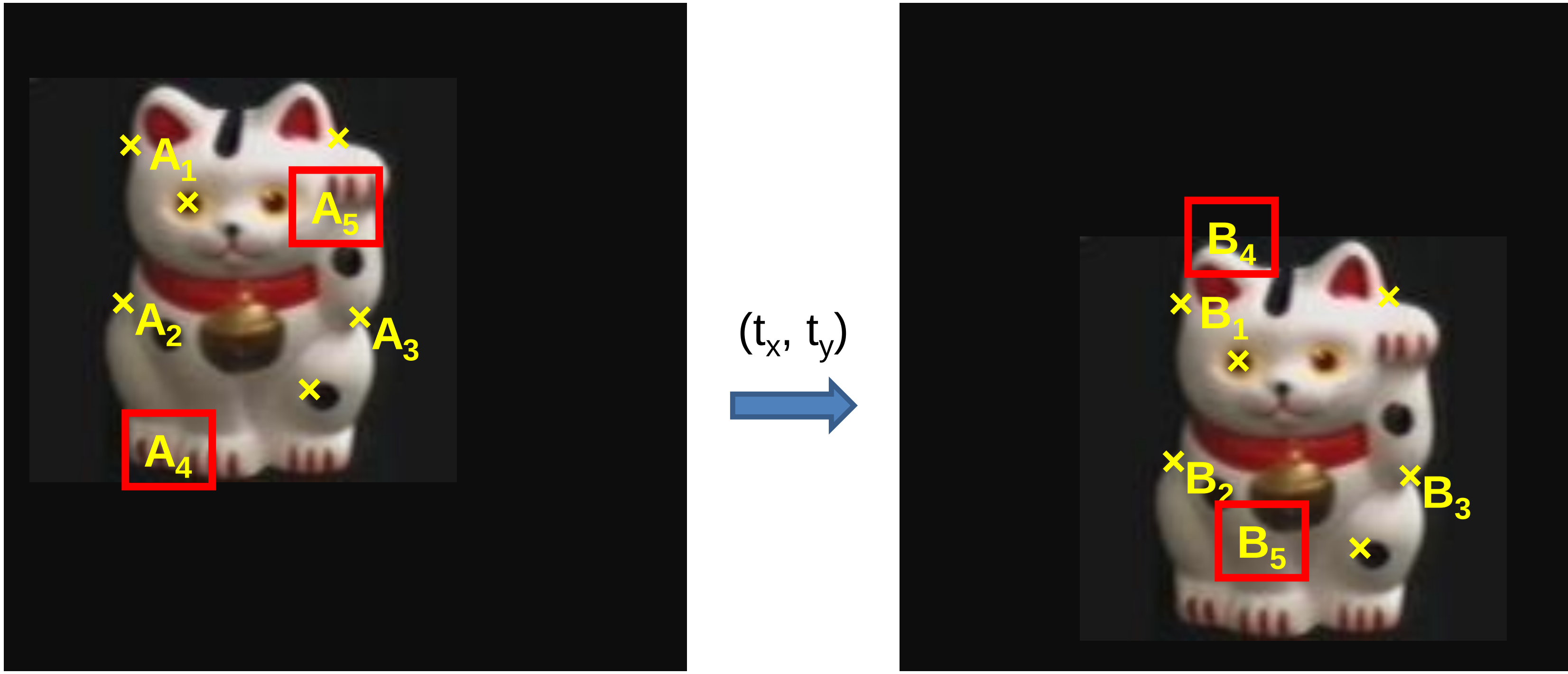
Repeat 1-3 until the best model is found with high confidence

Example: solving for translation

Given matched points in {A} and {B}, estimate the translation of the object

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



Problem: outliers

RANSAC solution

1. Sample a set of matching points (1 pair)
2. Solve for transformation parameters
3. Score parameters with number of inliers
4. Repeat steps 1-3 N times

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

RANSAC conclusions

Good

- Robust to outliers
- Applicable for larger number of objective function parameters than Hough transform
- Optimization parameters are easier to choose than Hough transform

Bad

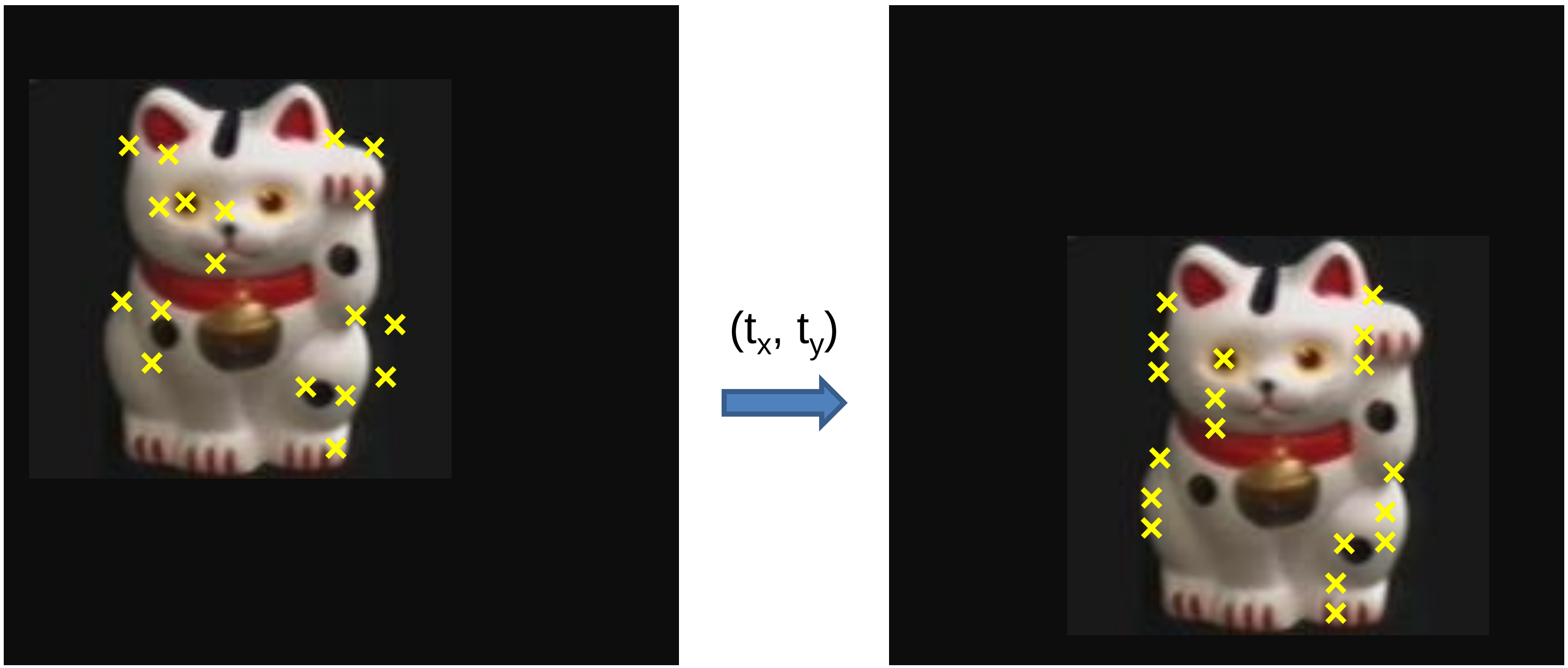
- Computational time grows quickly with fraction of outliers and number of parameters
- Not good for getting multiple fits

Common applications

- Computing a homography (e.g., image stitching)
- Estimating fundamental matrix (relating two views)

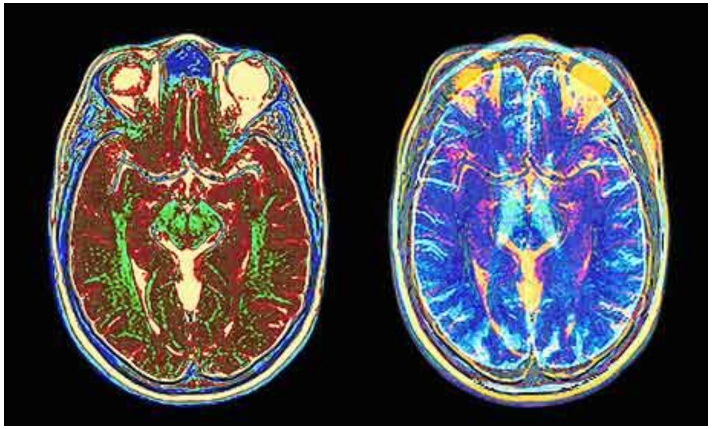
What if we want to align... but we have no matched pairs?

- Hough transform and RANSAC not applicable

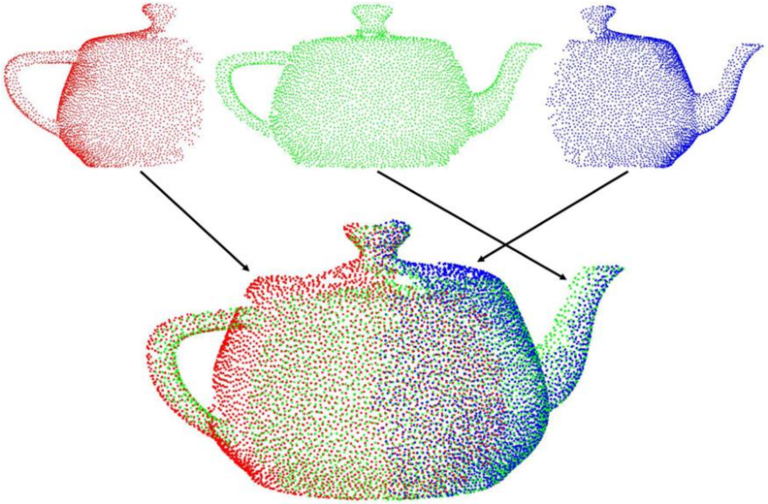


Problem: no initial guesses for correspondence

Important applications



Medical imaging: match brain scans or contours



Robotics: match point clouds

Kwok and Tang

Iterative Closest Points (ICP) Algorithm

Goal:

Estimate transform between two dense point sets S_1 and S_2

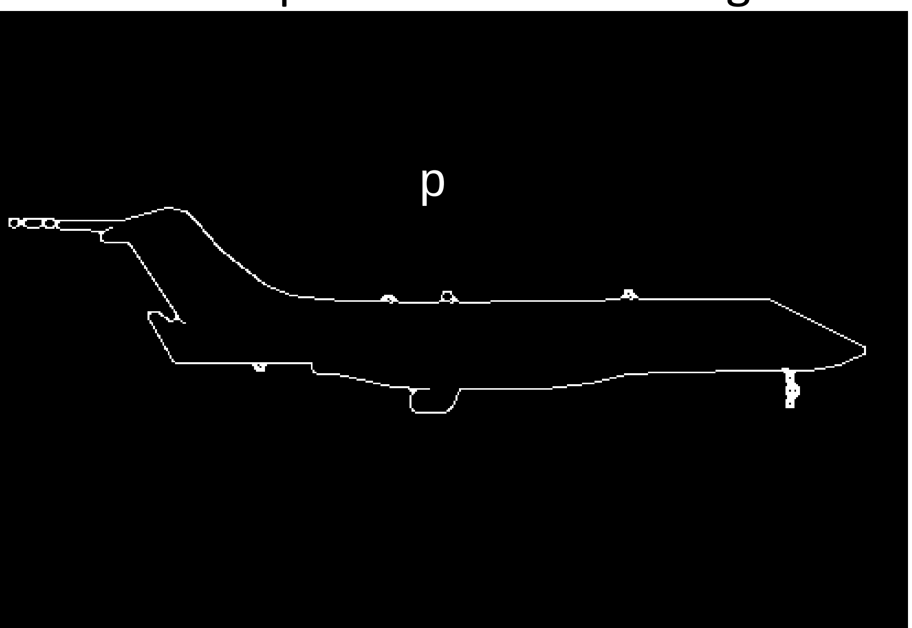
1. **Initialize** transformation
   - Compute difference in mean positions, subtract
   - Compute difference in scales, normalize
2. **Assign** each point in S_1 to its nearest neighbor in S_2
3. **Estimate** transformation parameters T
   - Least squares or robust least squares, e.g., rigid transform
4. **Transform** the points in S_1 using estimated parameters T
5. **Repeat** steps 2-4 until change is very small (convergence)

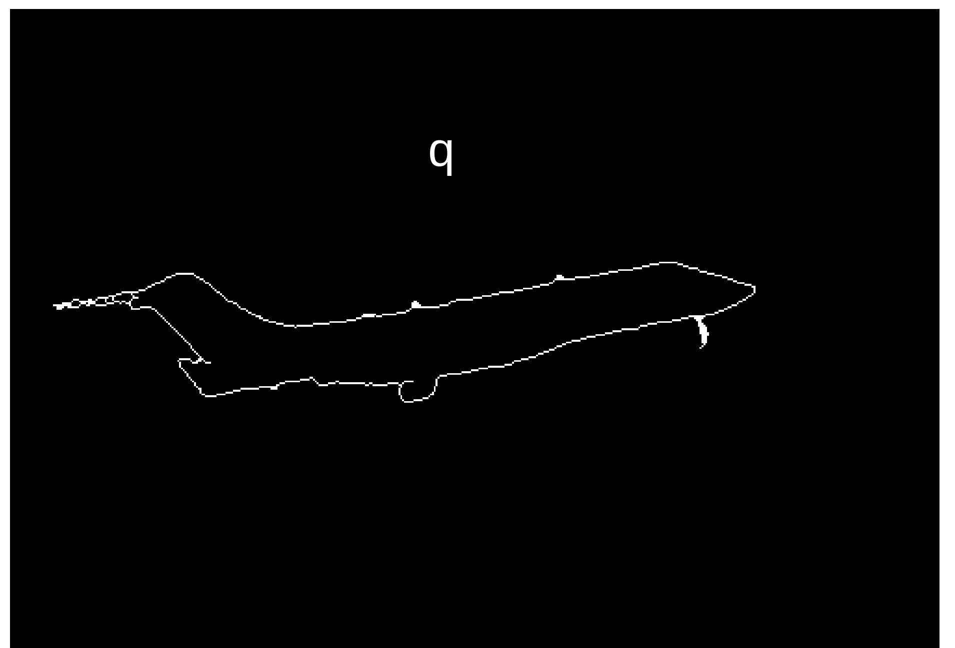
ICP demonstration

Bouaziz et al.

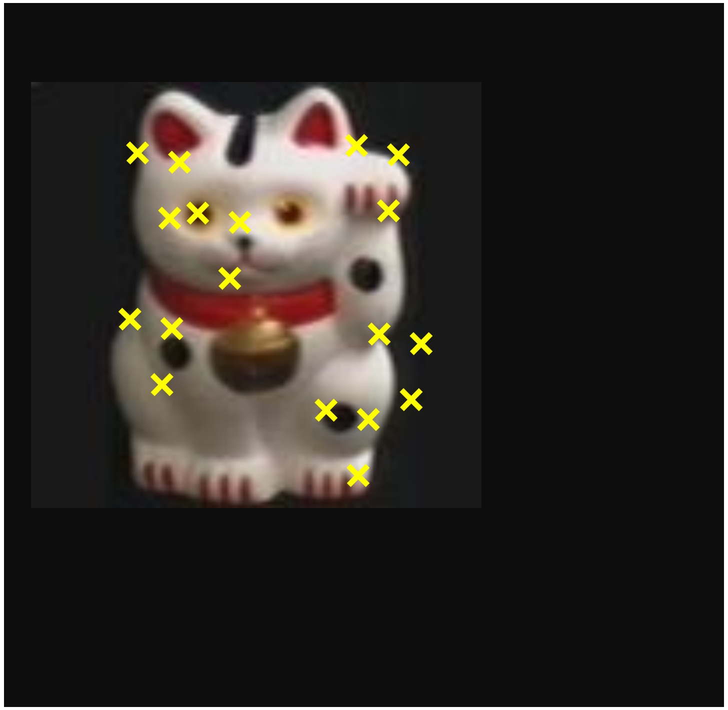
Example: aligning boundaries

1. Extract edge pixels $p_1 .. p_n$ and $q_1 .. q_m$
2. Compute initial transformation (e.g., compute translation and scaling by center of mass, variance within each image)
3. Get nearest neighbors: for each point p_i find corresponding $\text{match(i)} = \underset{j}{\text{argmin}}\, dist(pi, qj)$
4. Compute transformation ***T*** based on matches
5. Transform points ***p*** according to ***T***
6. Repeat 3-5 until convergence





Example: solving for translation



(t_x, t_y)

Problem: no initial guesses for correspondence

ICP solution

1. Find nearest neighbors for each point
2. Compute transform using matches
3. Move points using transform
4. Repeat steps 1-3 until convergence

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Sparse ICP

Sofien Bouaziz   *Andrea Tagliasacchi*   *Mark Pauly*

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Algorithm Summaries

- Least Squares Fit
  - Closed form solution
  - Robust to noise
  - Not robust to outliers
- Robust Least Squares
  - Improves robustness to outliers
  - Requires iterative optimization
- RANSAC
  - Robust to noise and outliers
  - Works with a moderate number of parameters (e.g, 1-8)
- Iterative Closest Point (ICP)
  - For local alignment only: does not require initial correspondences
  - Sensitive to initialization
- Hough transform
  - Robust to noise and outliers
  - Can fit multiple models
  - Only works for a few parameters (1-4 typically)

Feedback form

- Will send out later today
- 3 simple questions – good / bad / you.
- Please fill it in.