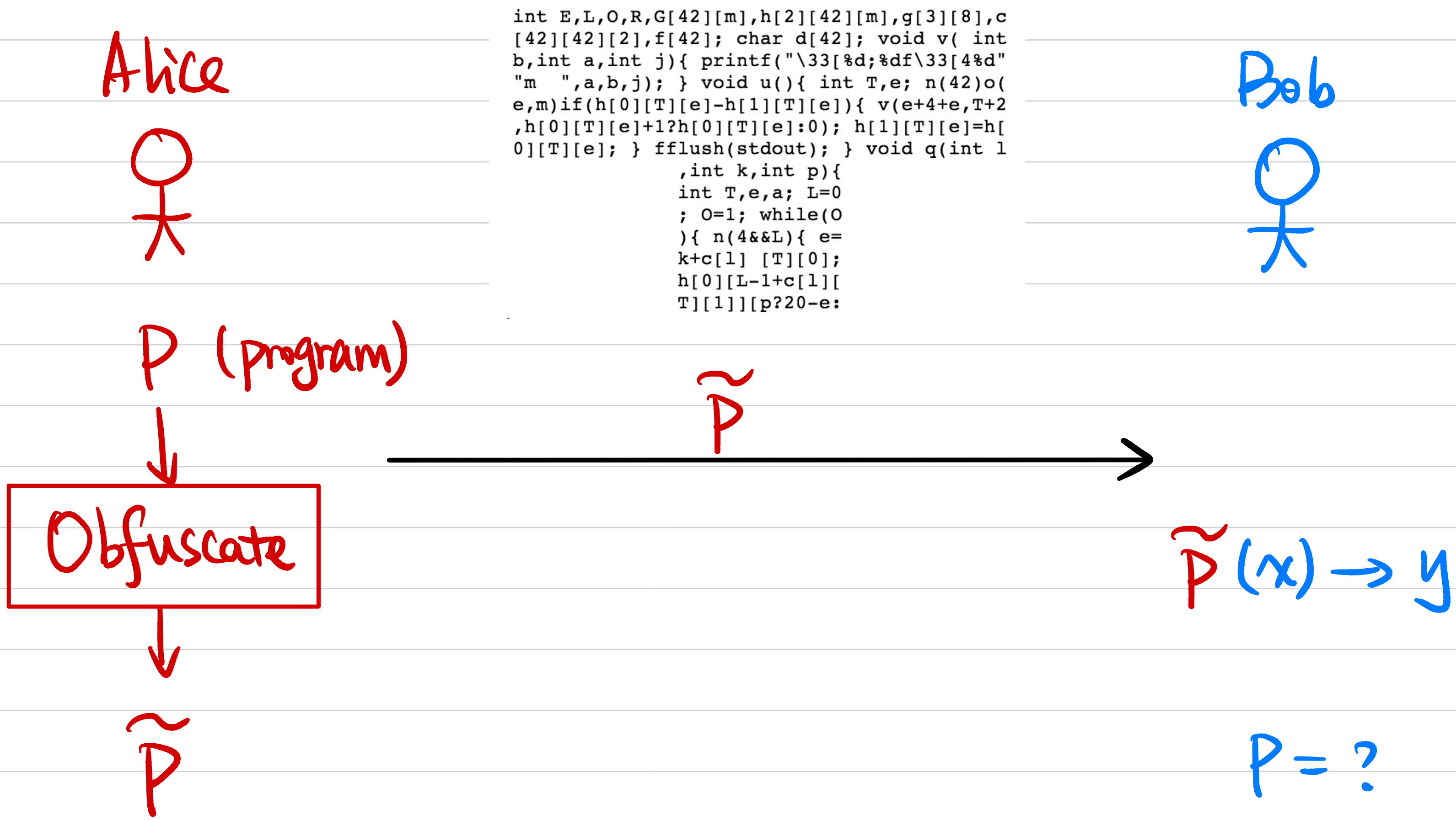
CSCI 1510

- Program Obfuscation (continued)
- Final Review

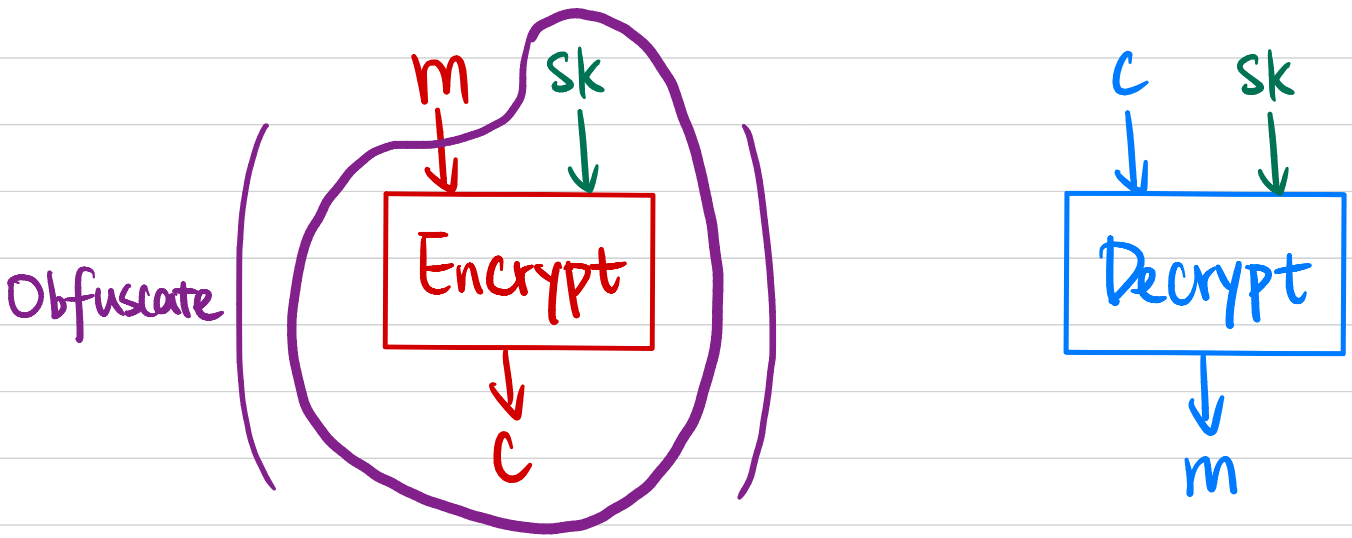
ANNOUNCEMENT: Course Feedback & Critical Review

Program Obfuscation



Goal: Make the program "unintelligible" without affecting its functionality.

Symmetric-Key to Public-Key

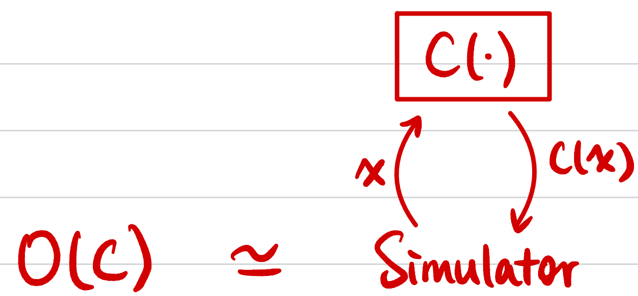


Formal Definition: Virtual Black Box (VBB)

Obfuscator O: $\quad C \xrightarrow{O} O(C)$

- **Functionality**: $O(C)$ computes the same function as C.
- **Polynomial Slowdown**: $|O(C)| \le poly(n) \cdot |C|$
- **Security (Virtual Black Box)**:

  $\forall$PPT A, $\exists$PPT S, s.t. $\forall C$, $\quad A(O(C)) \overset{c}{\simeq} S^{C(\cdot)}(1^{|C|})$.



Thm VBB obfuscator for all poly-sized circuits is impossible to achieve.

$$C(x) := \begin{cases} b & \text{if } x = a \\ m & \text{if } x(a) = b \\ 0 & \text{otherwise} \end{cases}$$

Run $O(C)\big(O(C)\big) \to m$

Formal Definition: Indistinguishability Obfuscation (iO)

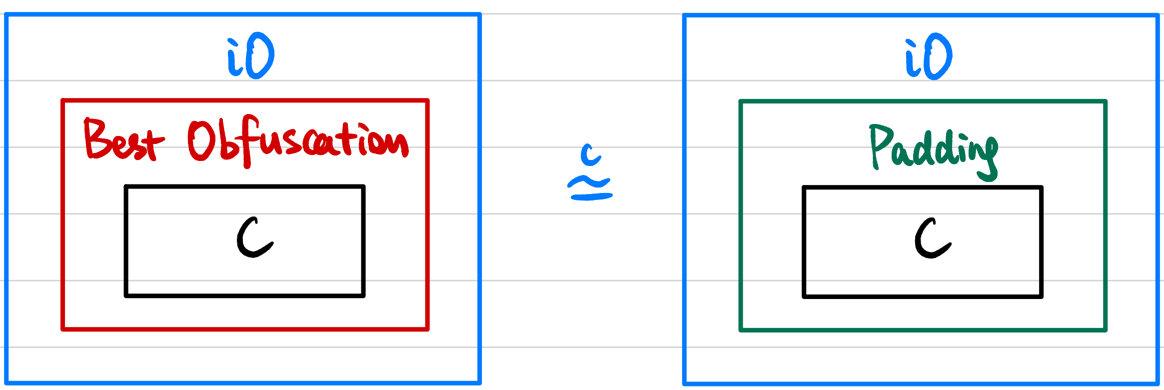
Obfuscator O: $C \xrightarrow{O} O(C)$

- **Functionality**: $O(C)$ computes the same function as C.
- **Polynomial Slowdown**: $|O(C)| \leq poly(n) \cdot |C|$
- **Security (indistinguishability obfuscation)**:

  If C_0 & C_1 compute the same function and $|C_0| = |C_1|$,

  then $O(C_0) \overset{c}{\approx} O(C_1)$

- **Best Possible Obfuscation**



PKE from iO

Let $G: \{0,1\}^n \to \{0,1\}^{2n}$ be a length-doubling PRG.

- $Gen(1^n)$:

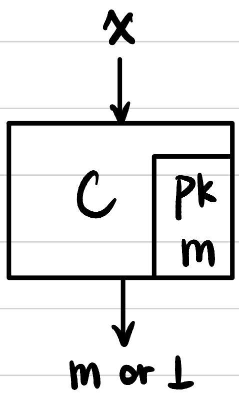
  $sk \xleftarrow{\$} \{0,1\}^n$

  $pk := G(sk)$

- $Enc_{pk}(m)$:

  $$C_{pk,m}(x) := \begin{cases} m & \text{if } G(x) = pk \\ \perp & \text{otherwise} \end{cases}$$

  Output $c \leftarrow iO(C_{pk,m})$



- $Dec_{sk}(c)$: $\quad C(sk) \to m$

Thm If G is a PRG and $iO(\cdot)$ is an idistinguishability obfuscator, then this PKE scheme is CPA-secure.

$\mathcal{H}_0$:

$sk \xleftarrow{\$} \{0,1\}^n$

$pk := G(sk)$

$$C_{pk,m}(x) := \begin{cases} m & \text{if } G(x) = pk \\ \perp & \text{otherwise} \end{cases}$$

Output $c \leftarrow i\mathcal{O}(C_{pk,m})$

↕ PRG

$\mathcal{H}_1$:

$pk \xleftarrow{\$} \{0,1\}^{2n}$

$$C_{pk,m}(x) := \begin{cases} m & \text{if } G(x) = pk \\ \perp & \text{otherwise} \end{cases}$$

Output $c \leftarrow i\mathcal{O}(C_{pk,m})$

↕ Stat. close

$\mathcal{H}_2$:

$pk \xleftarrow{\$} \{0,1\}^{2n}$

$C'_{pk,m}(x) := \perp$

Output $c \leftarrow i\mathcal{O}(C'_{pk,m})$

$$i\mathcal{O}(C'_{pk,m_0}) \stackrel{c}{\approx} i\mathcal{O}(C'_{pk,m_1})$$

Is it possible?

- 2001: Notion introduced
- 2013: Fist "Candidate" construction from multilinear maps
- 2013-2020: Attack, fixes, new constructions from new assumptions
- 2020: New construction from well-founded assumptions

Final Review

- Cryptographic Hardness Assumptions
  - Factoring / RSA Assumptions
  - DLOG / CDH / DDH Assumptions
  - LWE Assumption (Post-Quantum)

- Key Exchange
  - Definition
  - Construction: Diffie-Hellman

- Public-Key Encryption
  - Definition: CPA / CCA
  - Constructions: El Gamal / RSA / Regev

Final Review

- Theoretical Assumptions
  - One-Way Function / Permutation : Definition & Candidates
  - Hard-Core Predicate: Definition & Construction
  - PRG/PRF from OWP

  - Trapdoor Permutation: Definition & Candidate (RSA)
  - PKE from TDP

- Fully Homomorphic Encryption
  - Definition & Applications
  - Somewhat Homomorphic Encryption over Integers & from LWE (GSW)
  - Bootstrapping SWHE to FHE

Final Review

- Digital Signature
  - Definition
  - Hash-and-Sign Paradigm

  - Construction 1: RSA-FDH
  - Proof in the Random Oracle Model

  - Construction 2: Schnorr
  - Identification Scheme: Definition & Construction from DLOG (Schnorr)
  - Fiat-Shamir Transform

Final Review

- Zero-Knowledge Proof
  - Definition: Completeness / Soundness / Zero-Knowledge

  - Example: ZKP for Diffie-Hellman Tuples
  - Proof Technique: Rewinding

  - ZKP for All NP (Graph 3-Coloring)
  - Commitment Scheme

  - Non-Interaltive ZK

Final Review

- Secure Multi-Party Computation
  - Definition: Semi-Honest / Malicious
  - Applications
  - Example: Private Set Intersection from DDH
  - MPC for Any Function (GMW)
  - Oblivious Transfer: Definition & Construction from CDH

- Program Obfuscation
  - Definitions: VBB / iO
  - Example: PKE from iO

THANK YOU ^‿^