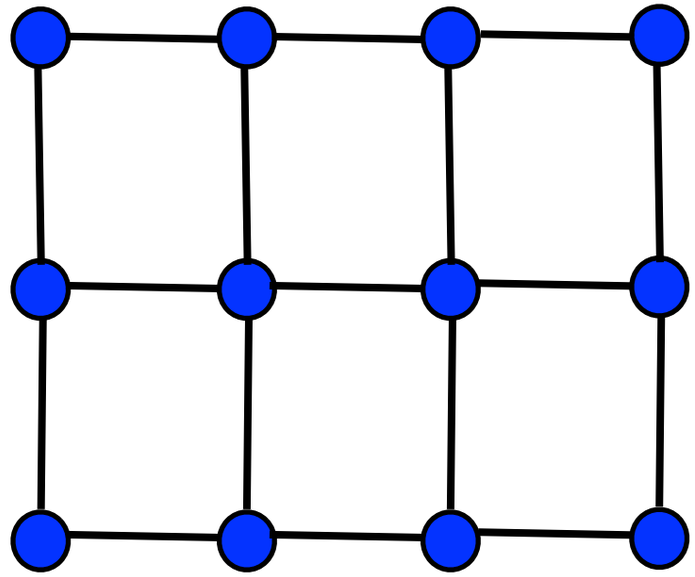
Machine learning 2013
Ising model
Image restoration

Ising model

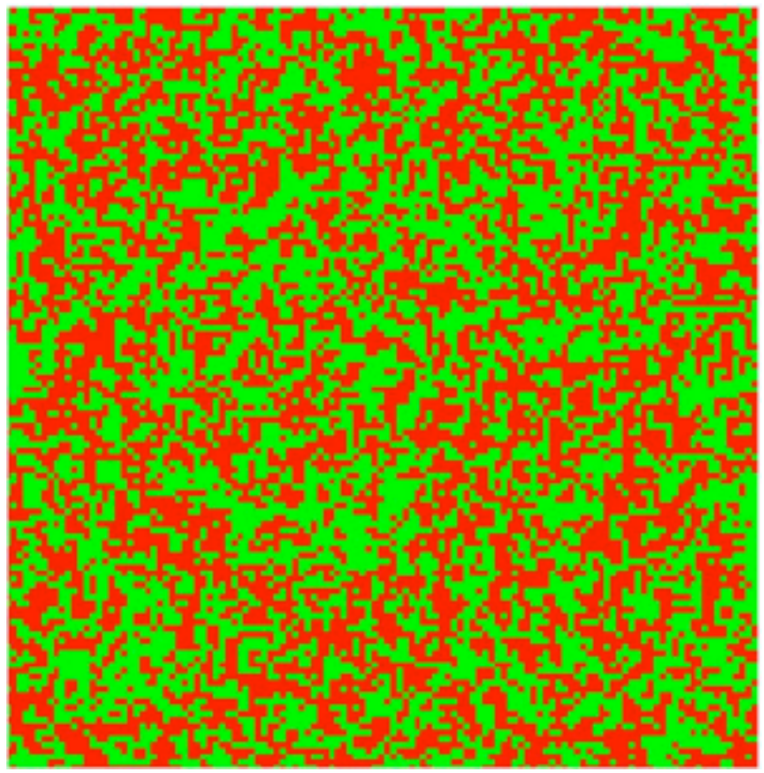
$$p(x) = \frac{1}{Z} e^{-E(x)}$$

$$E(x) = \beta \left(\sum_{(a,b) \in E} 1(x_a \neq x_b) \right)$$

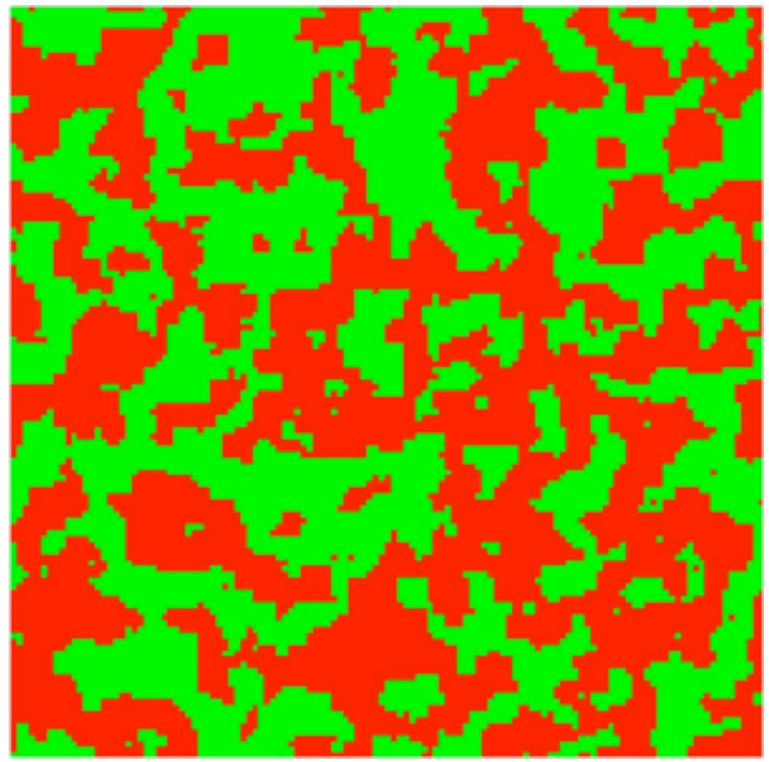


Ising model

samples from p(x)



small b

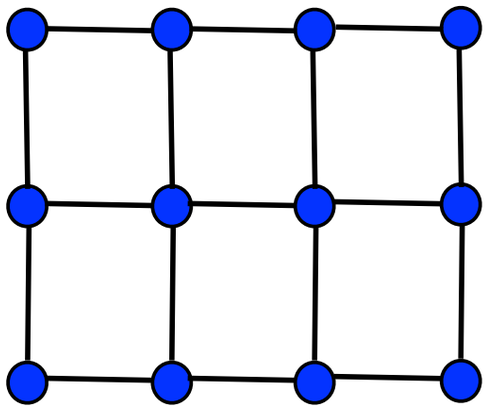


large b

sampling is hard - gibbs sampling: markov chain method

Gibbs samplimg

- Markov chain
- initialize x=0
- repeat
  - pick node a uniformly at random
  - sample new value for x_a from $p(x_a|x_{\setminus a})$
- x at time t > T comes from a distribution close to p(x)



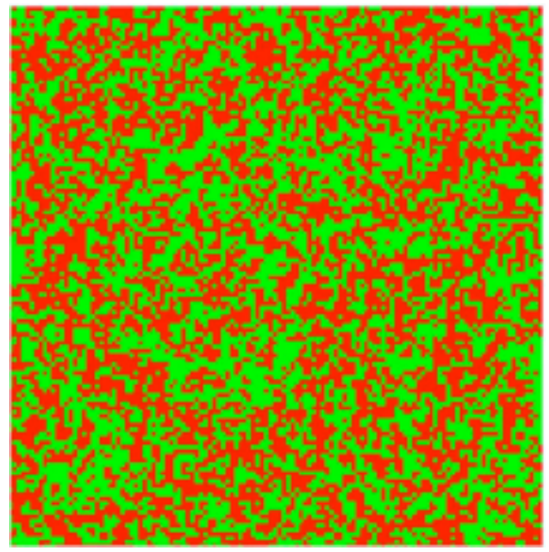


Image restoration

noisy image (y)

(c)

small b

(f)

large b

maximizing p(x|y) ~ p(y|x)p(x)

MAP Inference with Ising

$$p(x|y) = \frac{p(y|x)p(x)}{p(y)}$$

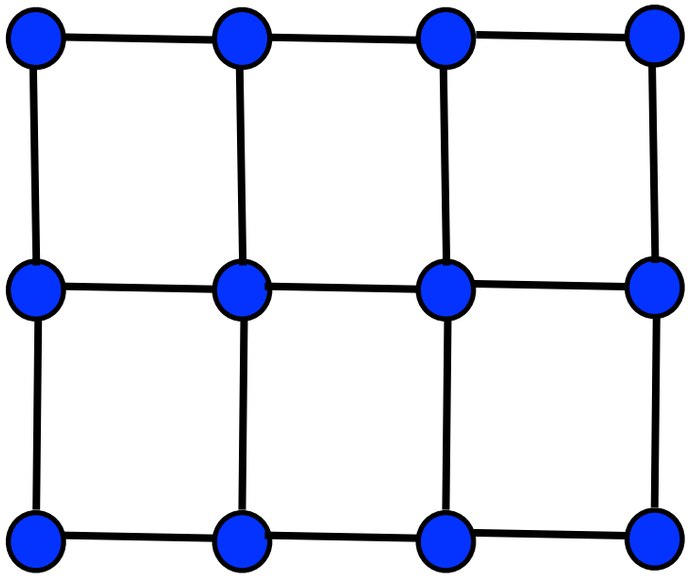
$$p(y|x) = \prod_a p(y_a|x_a) \qquad \log(p(y|x)) = \sum_a \log(p(y_a|x_a))$$

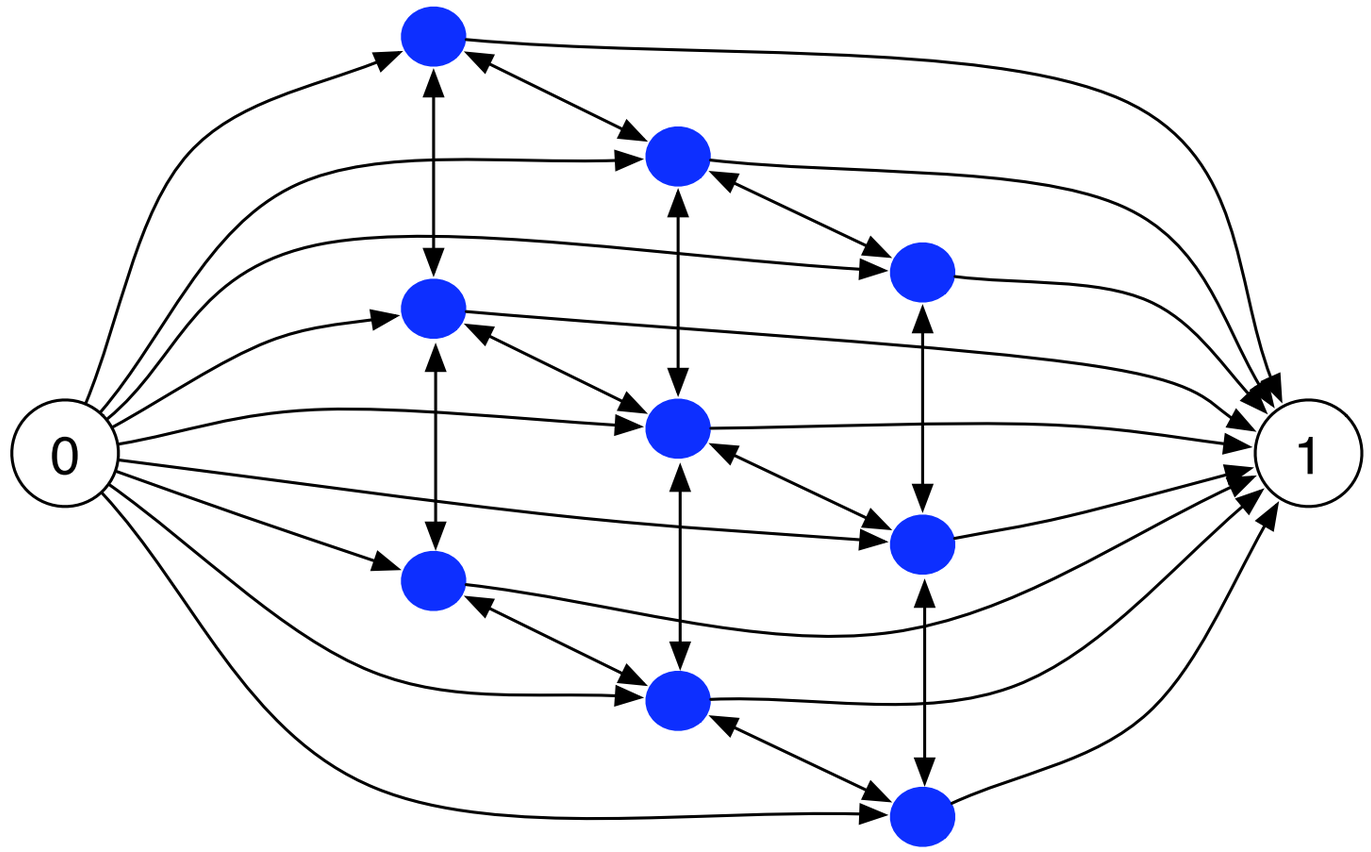
$$p(x) = \frac{1}{Z} e^{-E(x)} \qquad \log(p(x)) = -E(x) - \log Z$$

$$\log(p(x|y)) = \sum_a \log(p(y_a|x_a)) - \sum_{(a,b)\in E} \beta 1(x_a \neq x_b) - \log Z - \log(p(y))$$

$$E'(x) = \sum_a D_a(x_a) + \sum_{(a,b)} V(x_a, x_b)$$

Graph cuts



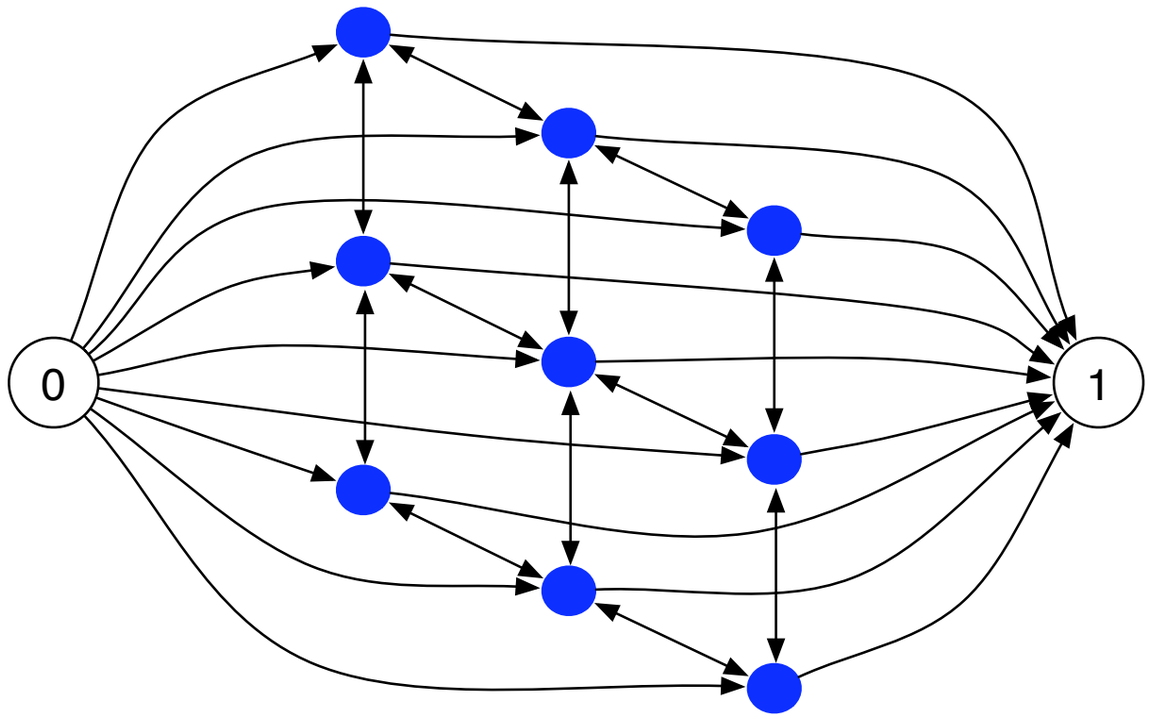


directed graph

s-t cut is a partition of the nodes (S,T) s in S, t in T

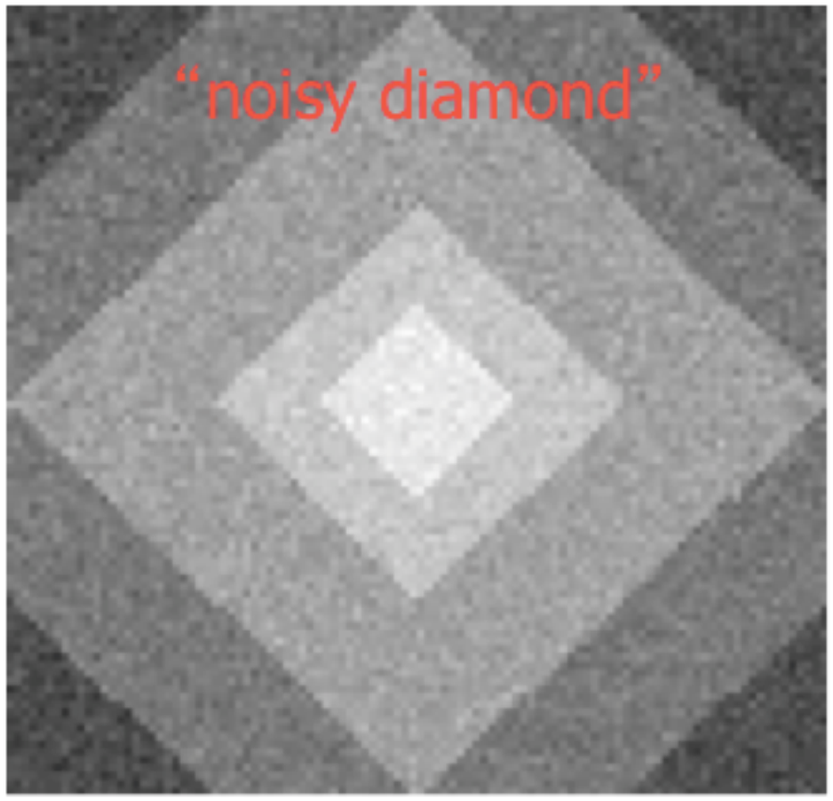
$$\mathrm{cut}(S,T) = \sum_{a \in S, b \in T} c(a,b)$$

Min-st cuts

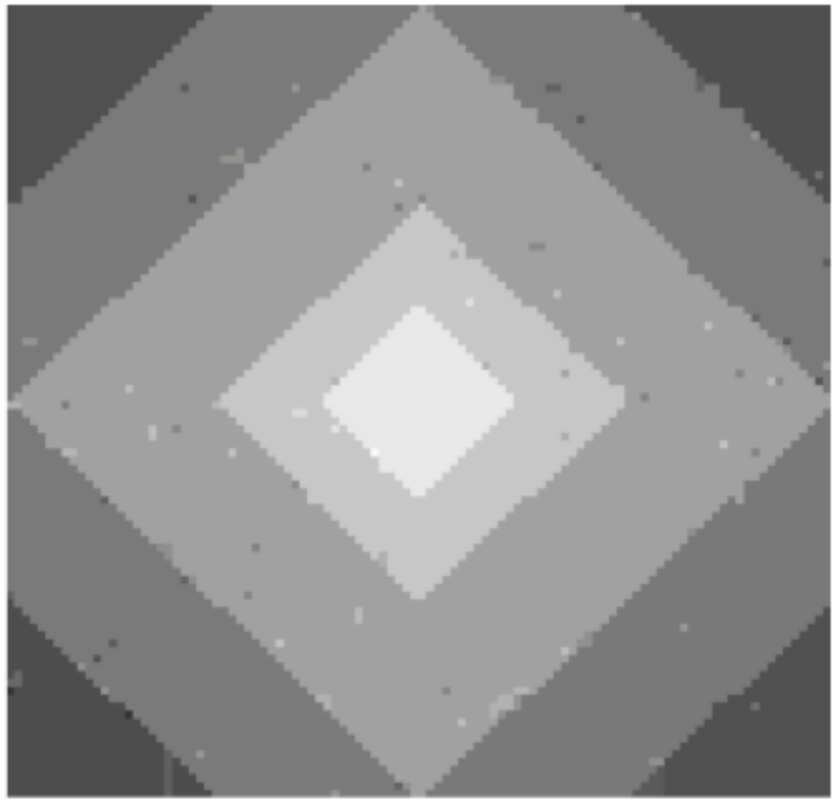


- Compute max-flow
  - start with f = 0
  - repeatedly find augmenting paths
- Retrieve min-cut
  - BFS from s until saturated edges

Image restoration



y



x

$$x, y \in \{0, 1, 2, \ldots, 255\}$$

$$E(x) = \beta(\#\text{discontinuities})$$

iid noise

Generalizations

- Ising:
  - x = 0/1
  - attractive pairwise terms
    - 0=V(1,1)=V(0,0)< V(1,0)=V(0,1)=beta
- graph-cuts generalizes to
  - V(1,1)+V(0,0) < V(0,1)+V(1,0)
- x = 0,1,2,...