CSCI1950-J: Take-Home Exam 2

Out: March 10, 2011 (revised: March 16, to remove Problem 3)
Due: March 17, 2011 (in class)

This is a strictly non-collaborative assignment. You may only discuss the questions and answers with the course staff. You are permitted to use the textbook, but no external resources.

The writeup for this homework will be collected in class on the due date. All work should be typed, preferably in LaTeX.

Readings: Textbook §3.1, §3.2, §3.3, §3.4.3, §4.1.3, §4.1.4, §4.2.1, §4.2.3

Problem 1

Given n points in the plane, construct a simple polygon having them as its vertices.

(a) Show that $\Omega(n \log n)$ is a lower bound to running time.

(b) Design and analyze an algorithm to solve the problem. (Hint: modify Graham's scan.)

Problem 2

Let S be a set of n (possibly intersecting) unit circles in the plane. We want to compute the convex hull of S.

(a) Show that the boundary of the convex hull of S consists of a straight line segments and pieces of circles in S.

(b) Show that each circle can occur at most once on the boundary of the convex hull.

(c) Let S' be the set of points that are the centers of the circles in S. Show that a circle in S appears on the boundary of the convex hull if and only if the center of the circle lies on the convex hull of S'.

(d) Give an $O(n \log n)$ algorithm for computing the convex hull of S.

Problem 3

It was brought to our attention that a complete solution to this problem appears in the textbook. We're removing it from the exam.

Problem 4

Let f be a function from $\{0, \ldots, 2^k - 1\} \times \{0, \ldots, 2^k - 1\}$ to $\{x \colon x \in \mathbb{R},\ x \geq 0\}$. Given that

$$P \triangleq \{(x, y, f(x, y)) \colon x \in \{0, \ldots, 2^k - 1\},\ y \in \{0, \ldots, 2^k - 1\}\}$$

is a set of $n = 4^k$ points in 3-space, design and analyze an $O(n)$-time algorithm to identify the maxima of P.