CSCI1950-J: Take-Home Exam 3

Out: April 7, 2011
Due: April 14, 2011 (in class)

This is a strictly non-collaborative assignment. You may only discuss the questions and answers with the course staff. You are permitted to use the textbook, but no external resources.

The writeup for this homework will be collected in class on the due date. All work should be typed, preferably in LaTeX.

Problem 1 (25 points)

Let $S \triangleq \{(0,8),(1,0),(2,3),(3,4),(4,5),(5,2),(6,9),(7,6),(8,7),(9,1)\}$.

(a) In different colors, draw the Voronoi diagram of S and a Delaunay triangulation of S.

(b) Draw the farthest-point Voronoi diagram of S.

Problem 2 (25 points)

You are trapped in the middle of a flat island full of hostile velociraptors. Fortunately, they are wearing GPS trackers and all seem to be sleeping. Your goal is to escape without waking them up.

(a) Design, analyze, and prove correct an efficient algorithm to find a maximally safe route off the island. Formally, you are given a set S of sites in the Euclidean plane where there are sleeping velociraptors. Your algorithm should find, as a sequence of line segments, a path from the origin to the point at infinity that maximizes the minimum distance to a site.

(b) Now suppose that you have a large supply of tranquilizers. You can safely put one velociraptor at a time back to sleep but no more. Explain how to modify your algorithm to maximize the minimum distance to the second closest velociraptor.

Problem 3 (25 points)

Let S be a set of n sites and consider the problem of computing just the Voronoi polygon containing a site $p \in S$.

(a) Give an $\Omega(n \log n)$-time lower bound.

(b) Suppose that the polygon containing p has at most k vertices. Design, analyze, and prove correct an $O(nk)$-time algorithm.

Problem 4 (25 points)

The medial axis of a simple polygon P is the locus of points that are equidistant from at least two points of the boundary of P. Give the pseudocode of an algorithm for constructing in time $O(n \log n)$ the medial axis of an n-edge convex polygon.

Problem 5 (25 points)

The Gabriele graph $GG(S)$ of a set S of points is defined as follows. There is an edge $\overline{p_i p_j}$ between $p_i, p_j \in P$ if the disk that has $\overline{p_i p_j}$ as its diameter contains no point of S in its interior.

Prove that $GG(S)$ is obtained from $Delaunay(S)$ by removing each edge of $Delaunay(S)$ that does not intersect its dual Voronoi edge.