Fall Semester 2007

CSCI 2570 - Introduction to Nanocomputing

Assignment 04

Due: Thursday, October 11, 2007

1 Problem 1

Compute directly or estimate the probability that an integer chosen uniformly at random from the range $[1, 10^9]$ is divisible by at least one of 4, 6, and 7.

Hint: If you choose to estimte the probability, consider using the principle of Inclusion-Exclusion.

2 Problem 2

Suppose you are one of four finalists for the best-paper award at a conference. The decision has been made but the results have not been announced. You don't know anything about the relative quality of the submissions, so you rightly suppose each finalist has a priori probability 1/4 of winning. You are friends with the conference chair, but he/she refuses to tell you who won. Persistent, you point out that at least 2 of the other three did not win, so you ask the chair to name a person who lost, picking randomly among the losers other than you, if you were a loser. The chair agrees and names someone who lost.

- Given the new information, has your chance of winning gone up to 1/3?
- Suppose you know one of the other finalists, Alice. If the chair points to a loser other than Alice, has her probability of winning gone up to 3/8?

Hint: Construct two sample spaces, one for the outcome before the chair provides information and a second that incorporates both the original outcome and the information from the chair.

3 Problem 3

Consider an $(n, k, d)_2$ error-correcting code with k information bits and block length n and minimum distance d, d odd. It follows that if at most $(d-1)/2$ errors occur in transmission, the transmitted codeword can be reconstructed. Prove that any $(n, k, d)_2$ code must satisfy:

$$\frac{k}{n} \leq 1 - H\left(\frac{d-1}{2n}\right) + o(1)$$

(Compare to the $1 - H(p)$ bound for noisy binary symmetric channel capacity.) By "$+o(1)$", we mean there exists a function $f(n, k, d)$ such that f goes to zero as $\min(n, k, d)$ goes to infinity.

To obtain this result show that $\log \binom{n}{r} = nH(r/n) - O(\log n)$.

Hint:

- Consider using the sphere-packing argument.
- You may use Sterling's formula, namely, $n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n (1 + O(1/n))$