

Learning and Inference in Probabilistic Graphical Models

Undirected Graphical Models

Feb. 8, 2010

2/15: Reading

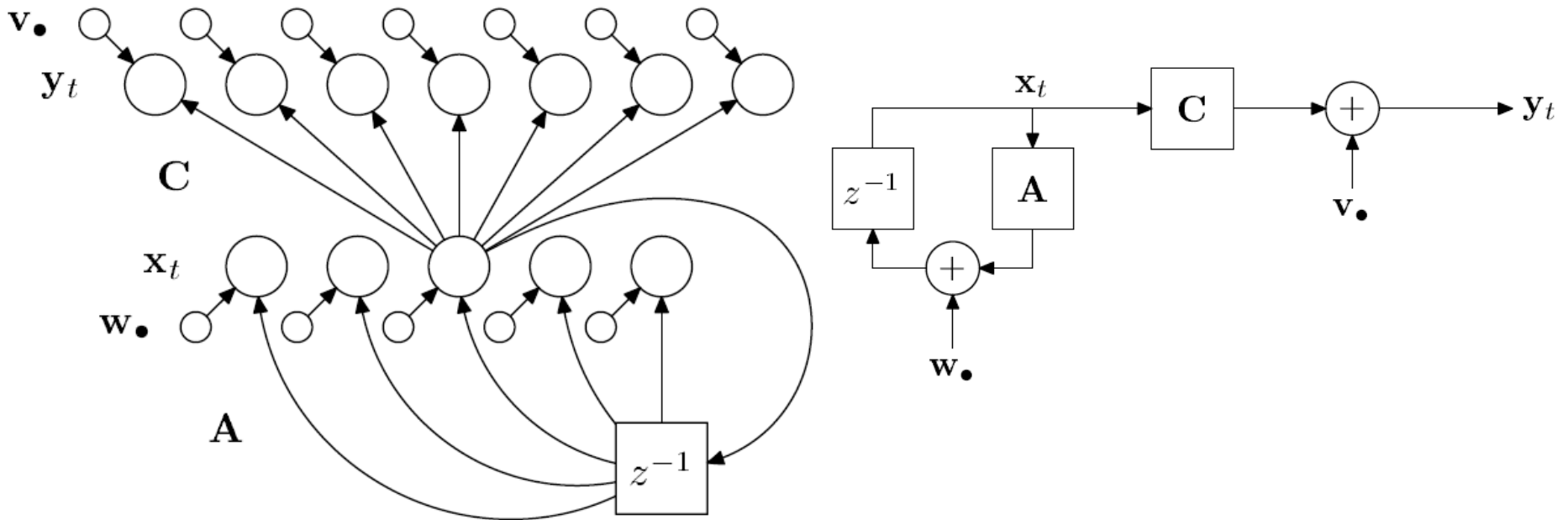
A Unifying Review of Linear Gaussian Models

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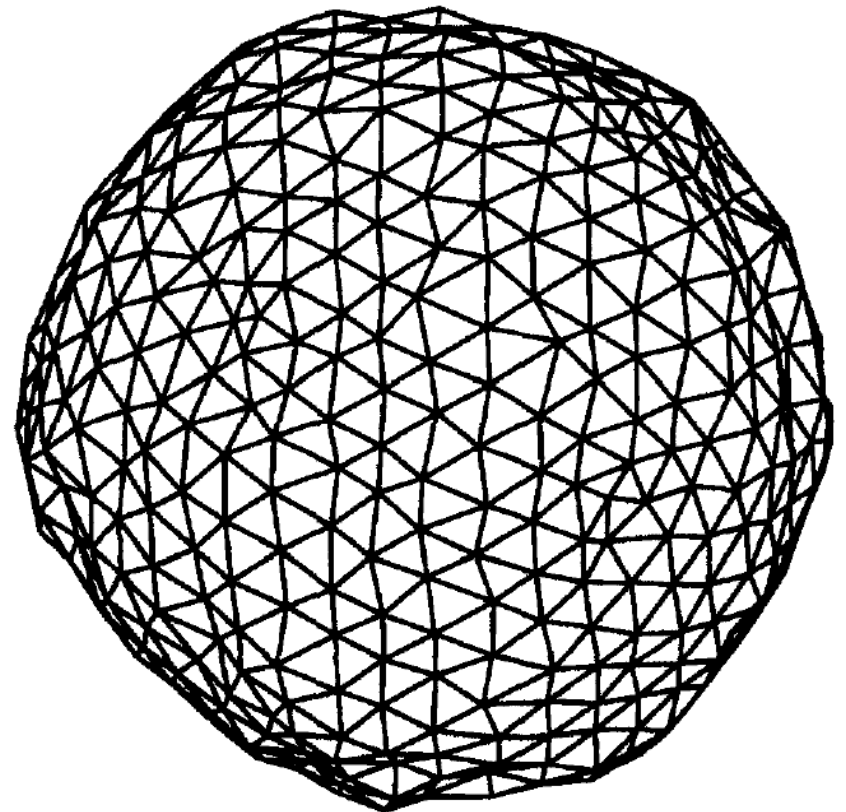
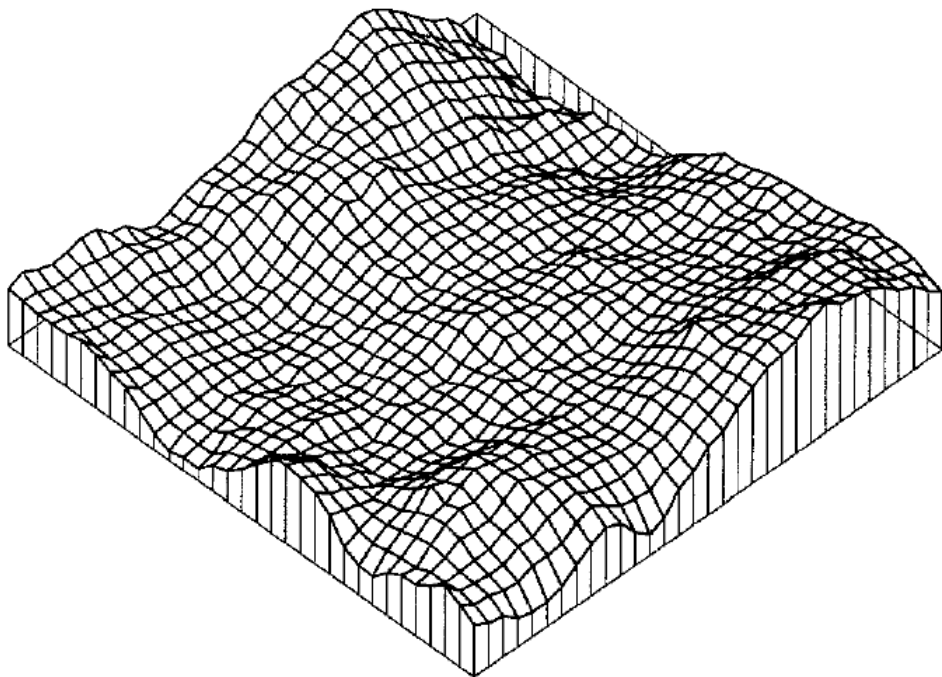


2/17: Reading

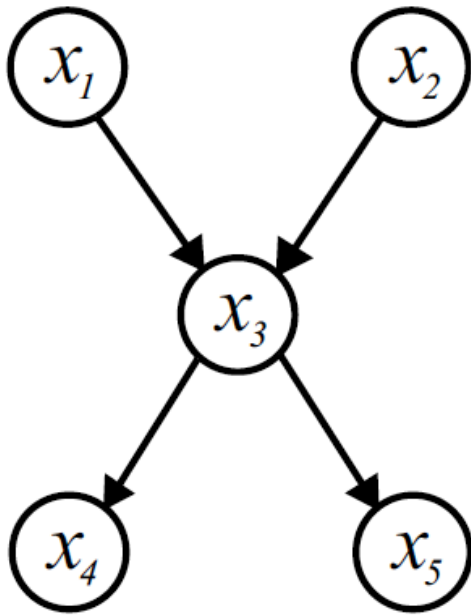
Bayesian Modeling of Uncertainty in Low-Level Vision

RICHARD SZELISKI

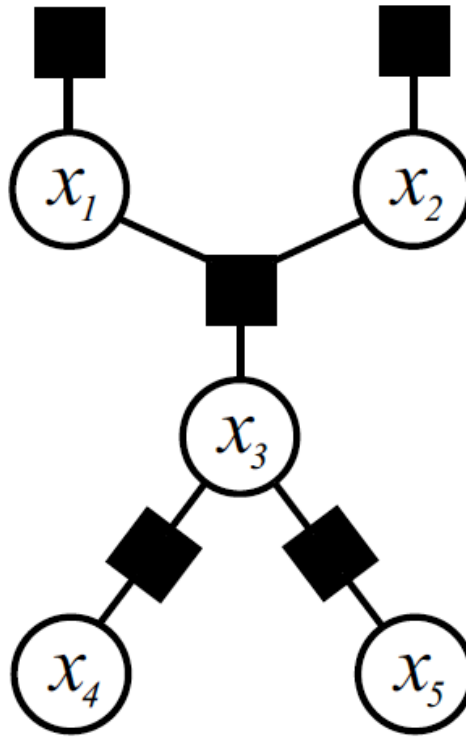
Digital Equipment Corporation, Cambridge Research Lab, One Kendall Square, Bldg. 700, Cambridge, MA 02139



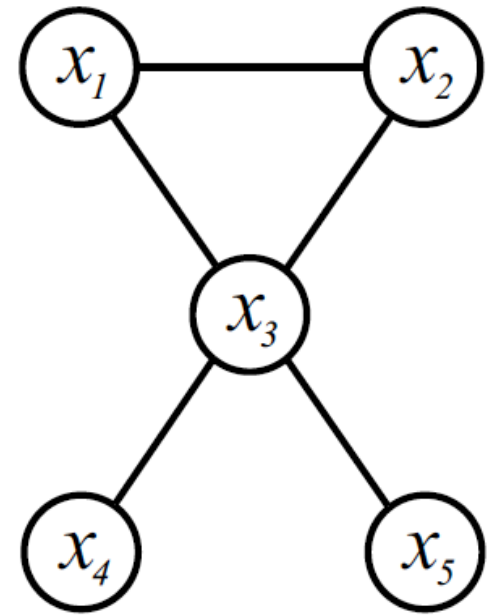
Types of Graphical Models



Directed

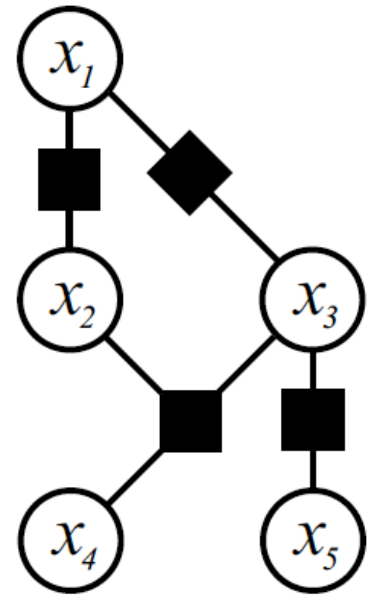
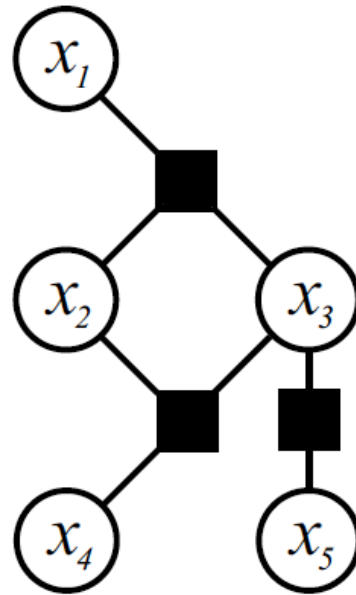
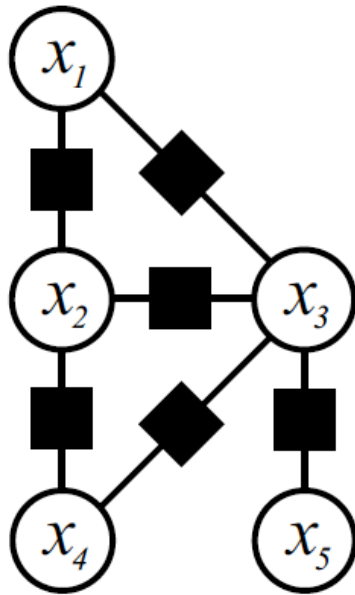
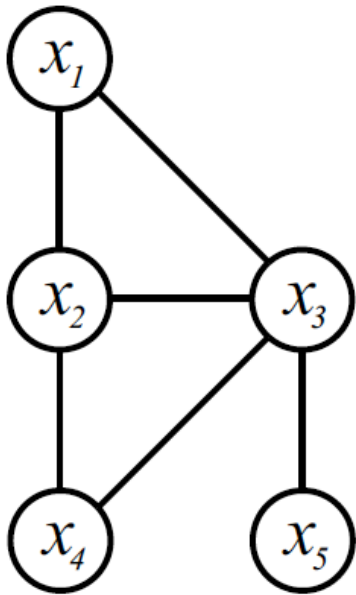


Factor

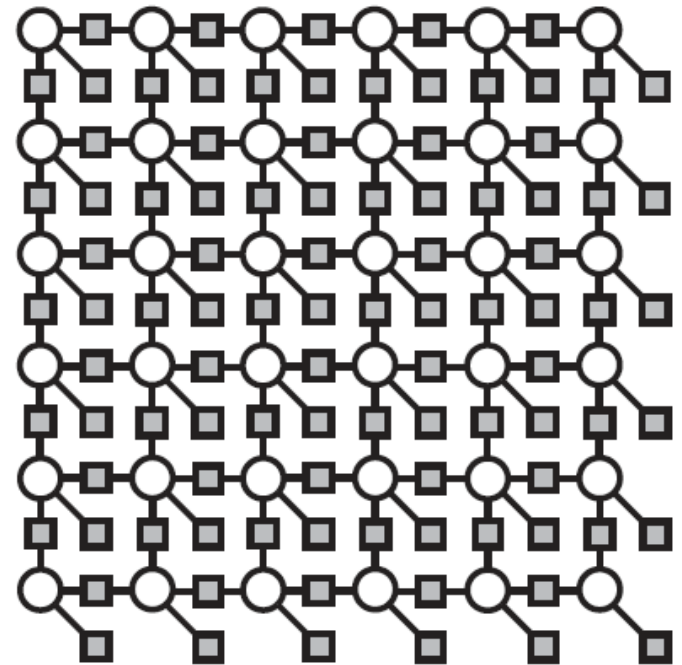
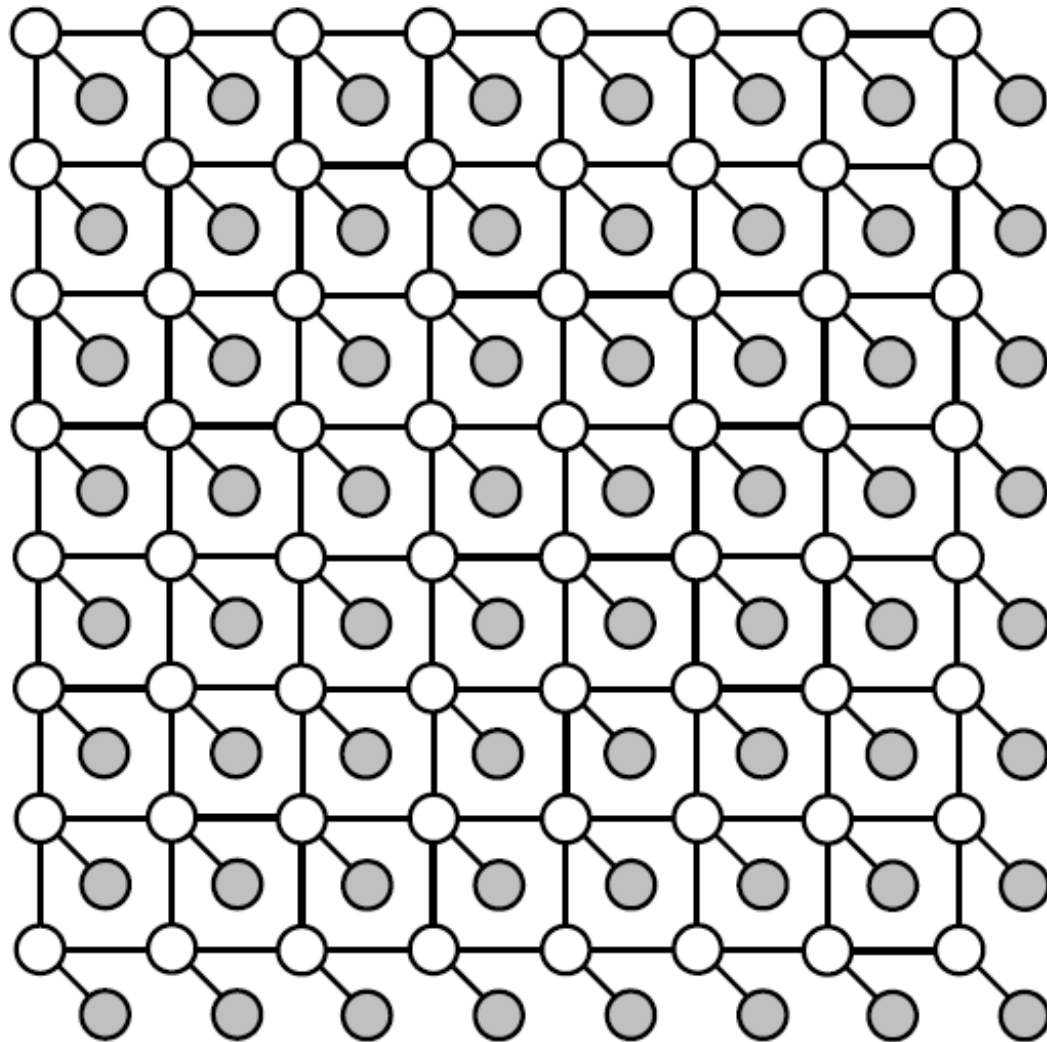


Undirected

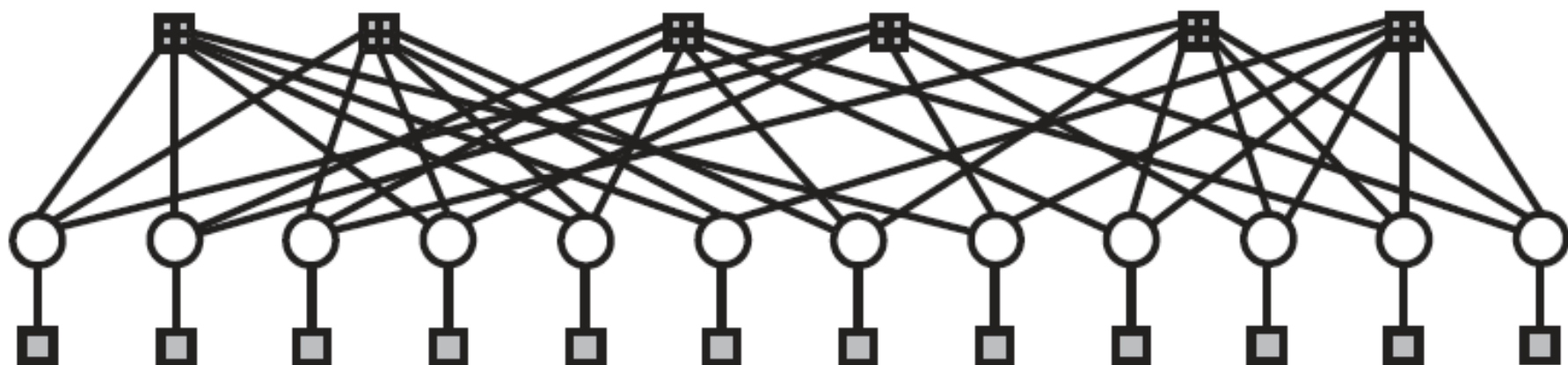
Fine-grained Factorization



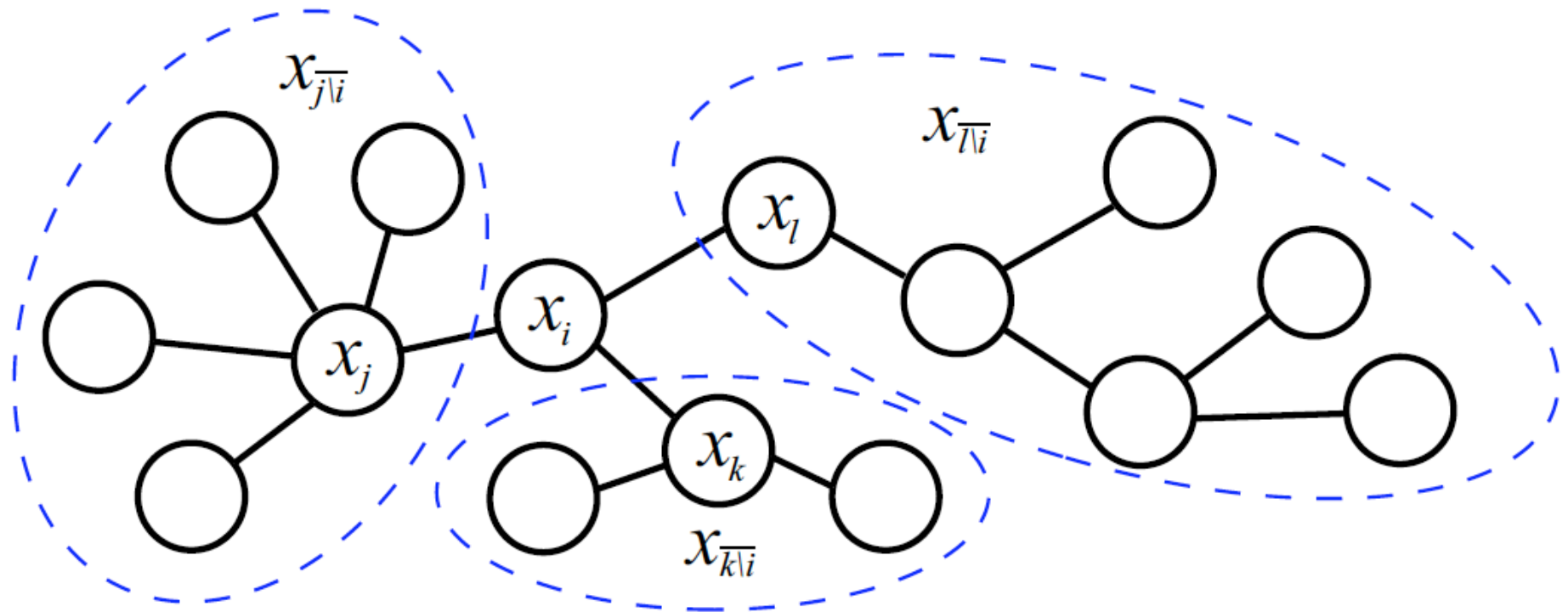
Pairwise Nearest-Neighbor MRF



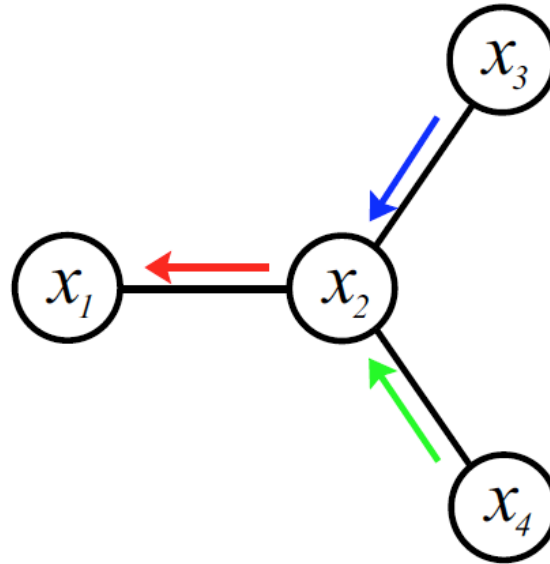
Low Density Parity Check (LDPC) Code



Markov Properties in Trees

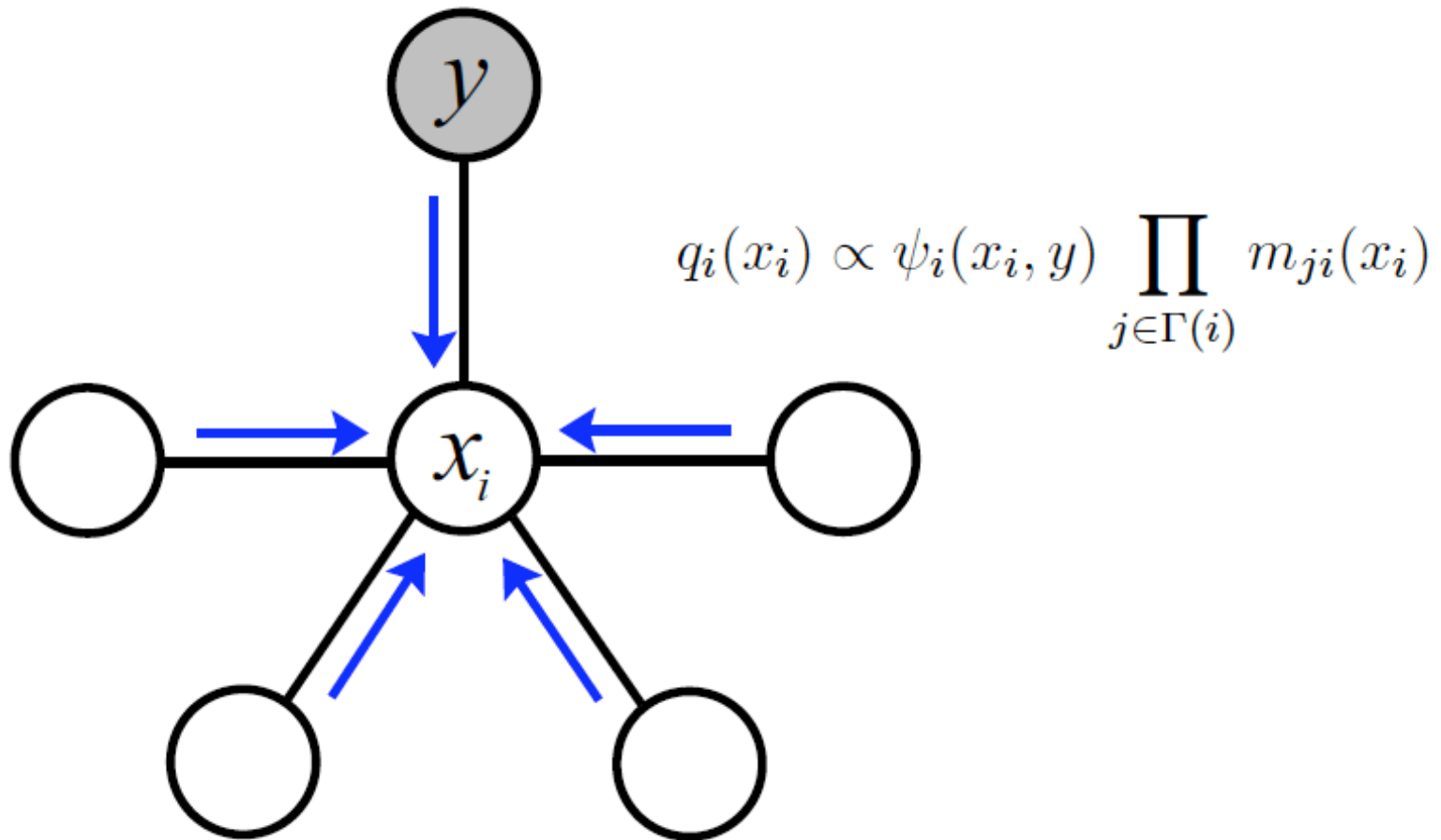


Message Passing

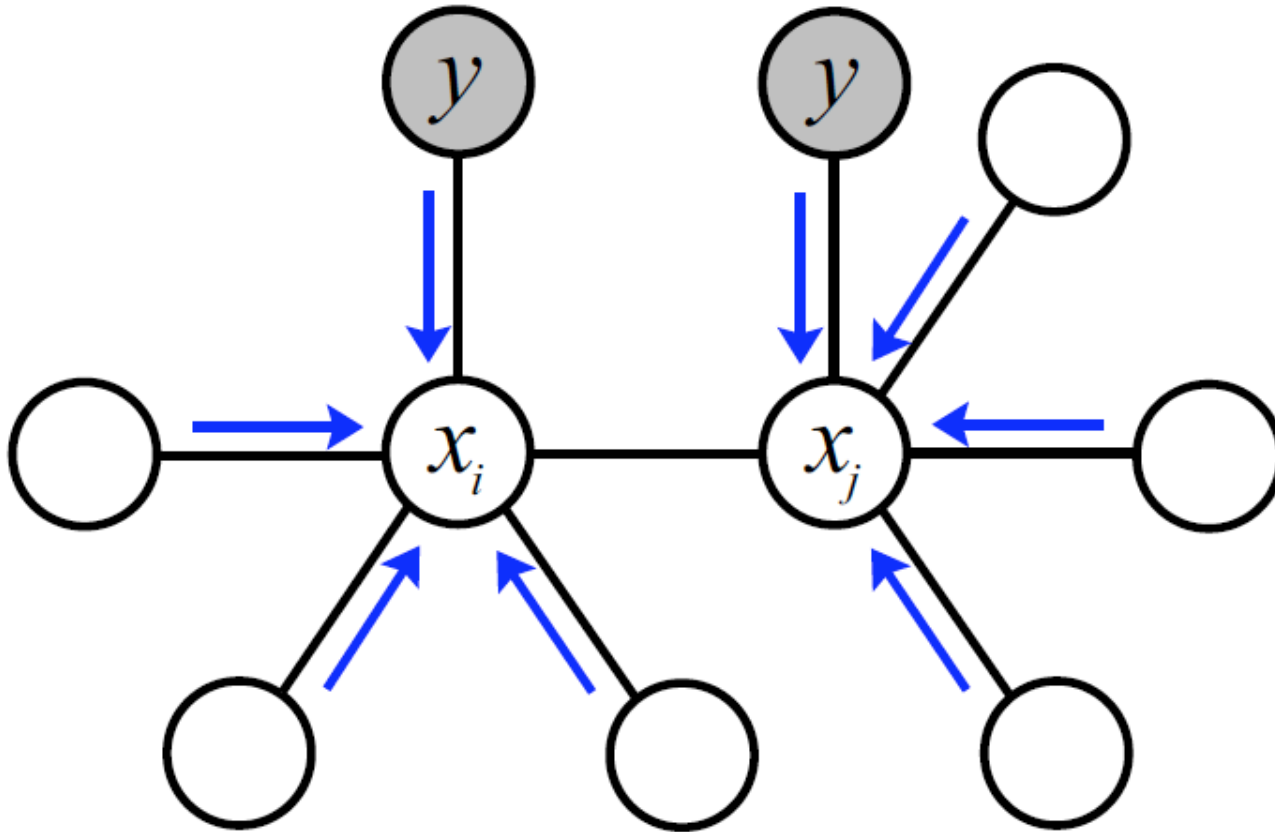


$$\begin{aligned}
 p(x_1) &\propto \iiint \psi_1(x_1)\psi_{12}(x_1, x_2)\psi_2(x_2)\psi_{23}(x_2, x_3)\psi_3(x_3)\psi_{24}(x_2, x_4)\psi_4(x_4) dx_4 dx_3 dx_2 \\
 &\propto \psi_1(x_1) \iiint \psi_{12}(x_1, x_2)\psi_2(x_2)\psi_{23}(x_2, x_3)\psi_3(x_3)\psi_{24}(x_2, x_4)\psi_4(x_4) dx_4 dx_3 dx_2 \\
 &\propto \psi_1(x_1) \int \psi_{12}(x_1, x_2)\psi_2(x_2) \left[\iint \psi_{23}(x_2, x_3)\psi_3(x_3)\psi_{24}(x_2, x_4)\psi_4(x_4) dx_4 dx_3 \right] dx_2 \\
 &\propto \psi_1(x_1) \int \psi_{12}(x_1, x_2)\psi_2(x_2) \underbrace{\left[\int \psi_{23}(x_2, x_3)\psi_3(x_3) dx_3 \right]}_{m_{32}(x_2)} \cdot \underbrace{\left[\int \psi_{24}(x_2, x_4)\psi_4(x_4) dx_4 \right]}_{m_{42}(x_2)} dx_2 \\
 &\quad \underbrace{\hspace{15em}}_{m_{21}(x_1)} \\
 m_{21}(x_1) &\propto \int \psi_{12}(x_1, x_2)\psi_2(x_2)m_{32}(x_2)m_{42}(x_2) dx_2
 \end{aligned}$$

BP Algorithm

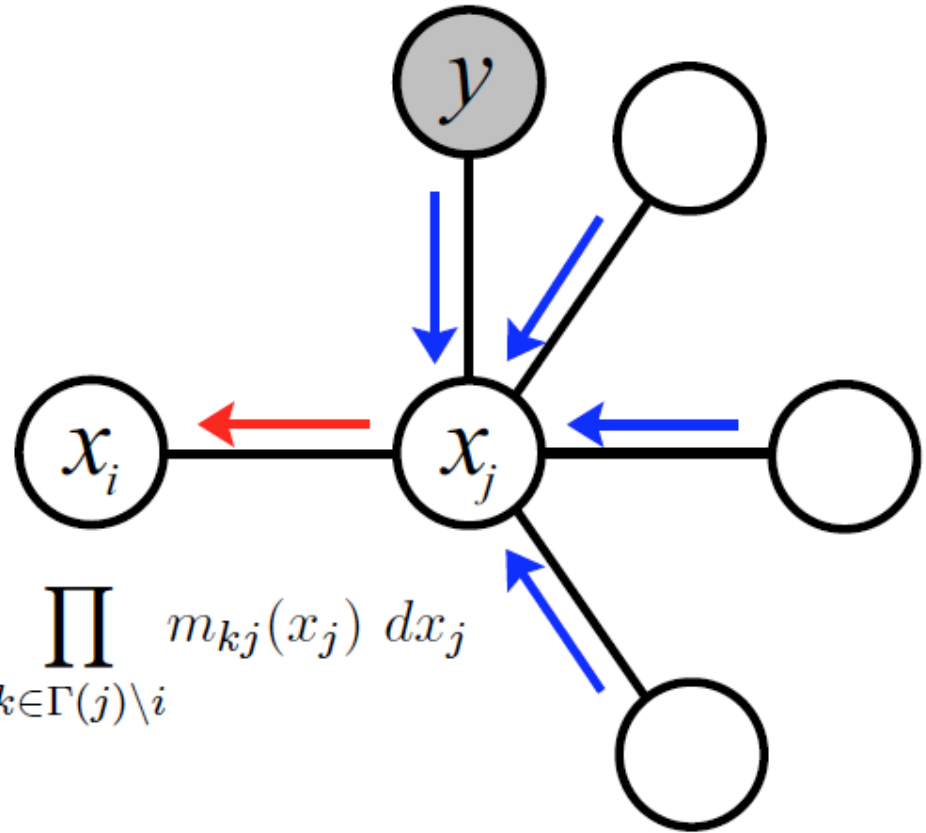


BP Algorithm



$$q_{ij}(x_i, x_j) \propto \psi_{ij}(x_i, x_j) \psi_i(x_i, y) \psi_j(x_j, y) \prod_{\ell \in \Gamma(i) \setminus j} m_{\ell i}(x_i) \prod_{k \in \Gamma(j) \setminus i} m_{kj}(x_j)$$

BP Algorithm



$$m_{ji}(x_i) \propto \int_{\mathcal{X}_j} \psi_{ij}(x_i, x_j) \psi_j(x_j, y) \prod_{k \in \Gamma(j) \setminus i} m_{kj}(x_j) dx_j$$