Tree Search

CSCI 2951-F

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A different view of how to plan

- So far, we have (mostly) assumed that we can compute a value function or policy in one big computation and use them for execution
- But search has classically been used in AI
- What if we emphasized search more?

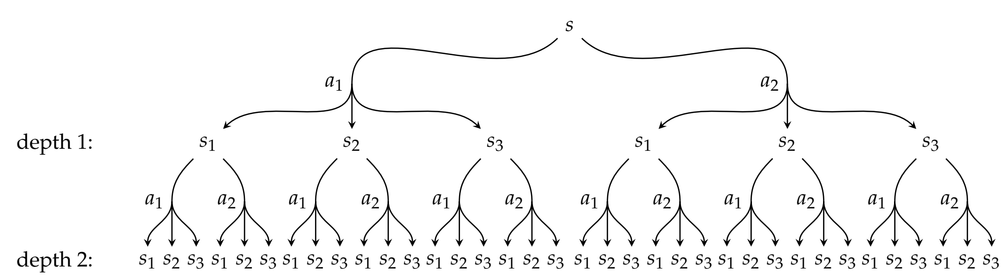

Figure 9.1. A forward search tree for a problem with three states and two actions.

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Searching before acting – “on line planning”

- Requires an accurate simulator
  - True for some robotics problems
  - Sensible assumption for most games
  - May or may not be reasonable in general
- Requires time to plan/search before each action - may not be practical for control problems
- Does not necessarily require planning for the entire state space, but
- Potentially wastes resources by continually replanning

Straw man

- Build a complete search tree out to depth d
- Alternate between action nodes and chance nodes
- Choose d so that $\gamma^d R_{max}$ is small
- Solve for policy in this tree recursively from leaves to root

- Problem:
  - b = branching factor = (#of actions x #possible next states)
  - b^d nodes

Branch and Bound

- Smarter approach:
- Maintain upper and lower bounds on the root utility
- Prune nodes of the search that are provably suboptimal
- Can be combined with heuristics, in which case A* is a special case

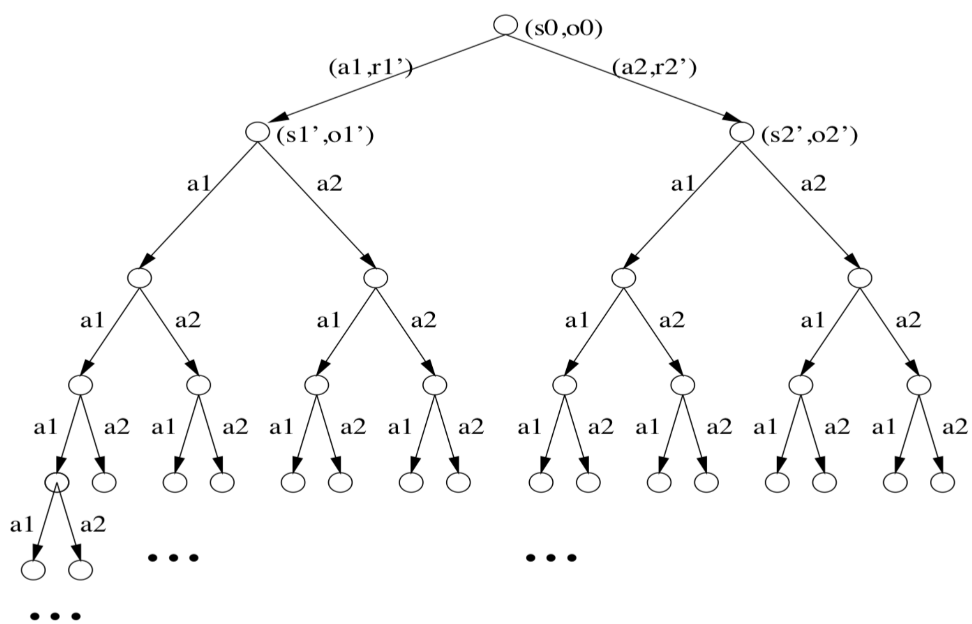
- Comment: This is a good approach that can save a lot of unnecessary searching in the best case, but don’t expect it to be a big win unless you have auxiliary information (heuristics) that are highly informative

Remove dependence on #next states

- Kearns et al. introduced trajectory trees
- Instead of considering all next states, sample next states
- Still branch on all actions
- Generate multiple trees instead one fat tree
- Evaluate potential policies against trees – value of policy is average value across trees

- Replaces dependence on #of next states with:
  - Dependence on VC dimension of policy space (linear), $1/\varepsilon^2$, $\log(1/\delta)$
  - # of trees needed to get good average evaluation of policies

Trajectory tree example



Kearns et al.

Trajectory tree limitations

- Main problem remains exponential dependence on d
- Each tree can still be very big
- Even if the number of trees isn't as bad as you might expect, may still very expensive to do in practice

A different approach: Bandits

- Bandit problem: Covered in more detail later
  - Multiple slot machines with unknown expected payoffs
  - Need strategy for playing arms so that learn which slot machine is best without too much opportunity cost of learning

- Regret: Difference between what you got and what you could have gotten if you played optimally

- Goal: Algorithms with bounded regret

UCB1

Deterministic policy: UCB1.
Initialization: Play each machine once.
Loop:

– Play machine j that maximizes $\bar{x}_j + \boxed{\sqrt{\frac{2 \ln n}{n_j}}}$, where $\bar{x}_j$ is the average reward obtained from machine j, n_j is the number of times machine j has been played so far, and n is the overall number of plays done so far.

Exploration bonus

From Auer et al., who show that UCB1 has regret **logarithmic in n**

Application to online planning

- Since we are using a simulator, we don't care so much about regret
- BUT: Don't still don't want to waste time
- Idea: What if we view each state as a sort of bandit problem when we explore a tree of possible outcomes from our current state?

Generic Monte Carlo Tree Search

```
function MonteCarloPlanning(state)
repeat
    search(state, 0)
until Timeout
return bestAction(state,0)

function search(state, depth)
if Terminal(state) then return 0
if Leaf(state, d) then return Evaluate(state)
action := selectAction(state, depth)
(nextstate, reward) := simulateAction(state, action)
q := reward + γ search(nextstate, depth + 1)
UpdateValue(state, action, q, depth)
return q
```

From Kocsis & Szepsesvari

Understanding UpdateValue

- Update value computes average value of descendants in the tree
- UCT includes an exploration bonus:
  - $C\sqrt{\frac{logN(s)}{N(s,a)}}$
  - C = sqrt(2) for bandits
- Issues:
  - Unlike bandits, some updates can include “stale” values from children, i.e., value of a node should reflect value of acting optimally for node’s children, but we update as we learn, so child values may not be right
  - How do you pick C?
  - Memory

Staleness

- K&S show that for sufficiently large C, we will converge to the correct values and action at the root
- Intuition:
  - Eventually, the leaf values will start converging to the correct values
  - If C is big enough, then we'll get enough samples for parents of these nodes to converge, overwhelming errors from earlier iterations
  - Apply this idea inductively

How to pick C

- Not much practical guidance here
- In practice, this will need to be very large
- Why?
  - Leaf values still matter
  - May need exponential number of steps to find leaf values with high rewards
  - No inherent way around this

- In practice:
  - Make C big enough so that you burn all the time you have
  - Works better than it should in many cases

Memory

- What if you can't afford to maintain value estimates for every node you encounter?
- On modern computers, you can run out of memory very quickly!

- When you hit a node you don't want to store the value for:
  - "Rollout"
  - Forward simulate to the end of the horizon using the current or random policy, and use this value
  - Does this make sense?

Go

- Ancient game that involves placing black/white stones on a lattice
- 9x9, 13x13, 19x19 (standard) versions
- Surround other players stones to capture and remove from board
- Objective: Maximize number of stones of your color on the board

Why Go is hard

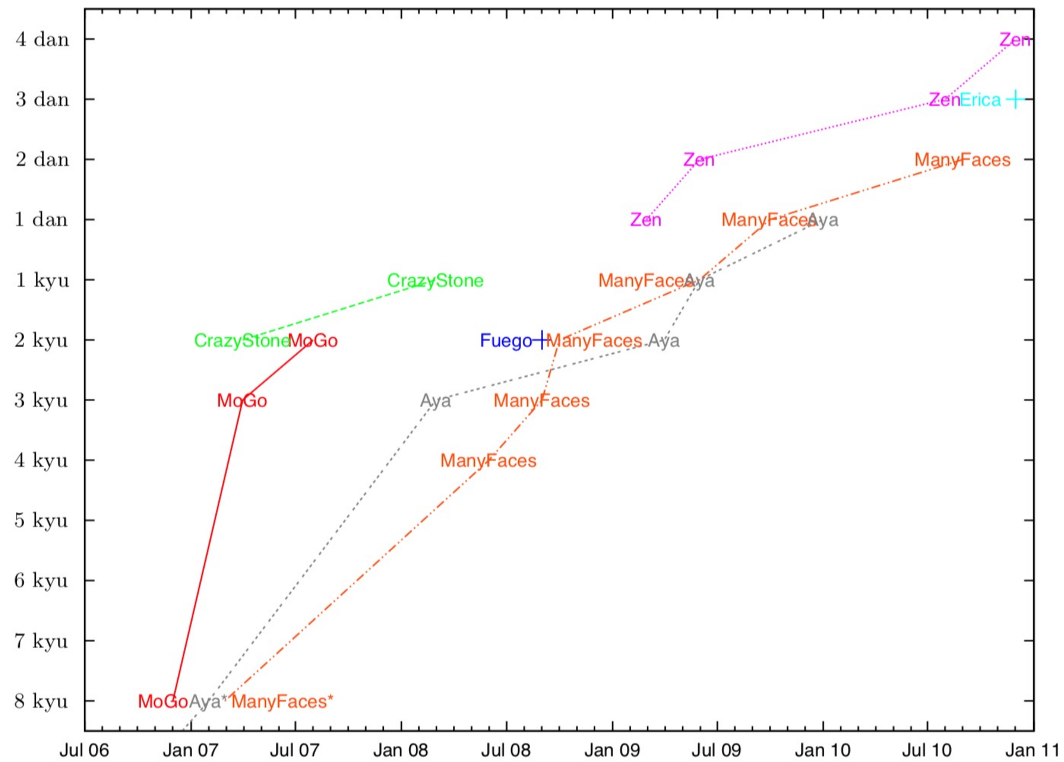
- ~200 moves per turn vs. ~37 in chess
- ~300 turns per game vs. ~57 in chess
- 10^{170} possible positions vs. 10^{47} in chess

- Evaluation is subtle – number of pieces on the board at any time is not in itself very predictive of outcome
- Very difficult to learn/invent a good evaluation function

MCTS for Go

- Classical approaches to Go did not do very well – nowhere close to master level play
- MCTS was a big improvement
- Tricks:
  - Parallelization
  - When/how to do rollouts
  - What policy to use for rollouts
  - Sharing information across subtrees
  - Using databases of expert moves when possible

Go Player ranking vs. time



From Gelly et al.

Relatively flat progress for decades until MCTS comes on the scene (2006), then rapid progress,

(AlphaGo defeats 9 dan Lee Sedol in 2016.)

Does this work for other games?

- Kind of, but not all

- Not a big win for chess (w/o additional tricks; see alphazero)

- What’s happening?
  - No practical way to pick C big enough to satisfy conditions for theoretical convergence to optimal behavior
  - Can’t explore the entire (remaining) tree except very close to end of game
  - Rollouts are very important for estimating the value of the truncated tree
  - Rollouts not reliable for games with important but narrow paths

Rollouts: Chess vs. Go speculation

- Go positions are hard to evaluate, but perhaps at a certain point, the good ones and bad ones have **wide paths** towards certain outcomes that are hard to miss with sampling

- Chess tends to have very **narrow paths**, so that even towards the end of the game, getting towards a particular outcome can be like threading a needle – hard to find with sampling

Search vs. Value Function Approximation

- In practice, this is a ***false dichotomy***
- Many practical approaches combine both:
  - Search to given depth
  - Use value function approximation for the state values at the leaves
- Discussion: Why does this combination make sense?