CS2951-S Homework 1
Due 4 March 2016
email pdf to `mph@cs.brown.edu`

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0.1 Exercise 2.5

Define the composition of a carrier map followed by a simplicial map. Prove that the composition is a carrier map. Moreover, if both are chromatic, their composition is chromatic.

0.2 Exercise 3.4

Let $\mathcal{A}$ and $\mathcal{B}$ be simplicial complexes, and $\Phi : \mathcal{A} \to 2^{\mathcal{B}}$ a rigid carrier map. Assume that $\mathcal{A}$ is pure of dimension d, and Φ is surjective, meaning that every simplex of $\mathcal{B}$ belongs to $\Phi(\sigma)$, for some $\sigma \in \mathcal{A}$. Prove that $\mathcal{B}$ is pure of dimension d.

0.3 Exercise 3.7

Let $\mathcal{A}$ and $\mathcal{B}$ be simplicial complexes, and $\Phi : \mathcal{A} \to 2^{\mathcal{B}}$ a surjective carrier map. Prove that if Φ is strict, then for each simplex τ of $\mathcal{B}$ there is a unique simplex σ in $\mathcal{A}$ of smallest dimension, such that $\Phi(\sigma)$ contains τ. Thus, if $\mathcal{B}$ is a subdivision of $\mathcal{A}$ with carrier map Φ, the *carrier* of a simplex in $\mathcal{B}$ is well defined.

0.4 Exercise 4.5

Here is another robot convergence task. In the *Earth Agreement* task, robots are placed at fixed positions on (a discrete approximation of) the Earth, and must converge to nearby points on the Earth's surface.

The input complex is a 3-simplex $\tau^3 = \{0, 1, 2\}$ (the Earth), and the output complex is $\mathrm{skel}^2 \tau^3$ (the Earth's surface). The robots start at any of the four vertices of τ^3. If they all start on one or two vertices, each process halts on one of the starting vertices. If they start on three or more vertices, then they converge to at most three vertices (not necessarily the starting vertices). The task's carrier map is

$$\Delta(\sigma) = \begin{cases} \sigma & \text{if } \dim \sigma \leq 1 \\ \mathrm{skel}^2 \tau & \text{if } \dim \sigma > 1 \end{cases}$$

Show that there is a colorless single-layer immediate snapshot protocol for this task. Explain why this task is not equivalent to 3-set agreement with 4 input values.

Now consider the following variation. Let the output complex be $\mathrm{Div}\,\mathrm{skel}^2\,\tau$, where Div is an arbitrary subdivision. As before, the robots start at any of the four vertices of τ^3. If they start on a simplex σ of dimension 0 or 1, then they converge to a single simplex of the subdivision $\mathrm{Div}\,\sigma$. If they start on three or more vertices, then they converge to any simplex of $\mathrm{Div}\,\mathrm{skel}^2\,\tau^3$. This carrier map is

$$\Delta(\sigma) = \begin{cases} \mathrm{Div}\,\sigma & \text{if } \dim\sigma \leq 1 \\ \mathrm{Div}\,\mathrm{skel}^2\,\tau & \text{if } \dim\sigma > 1 \end{cases}$$

Show that there is a colorless immediate snapshot protocol for this task. (Hint: use the previous protocol for the first layer).

Let us change the carrier map slightly to require that if the processes start on the vertices of a 2-simplex σ, then they converge to a simplex of $\mathrm{Div}\,\sigma$. The new carrier map is

$$\Delta(\sigma) = \begin{cases} \mathrm{Div}\,\sigma & \text{if } \dim\sigma \leq 2 \\ \mathrm{Div}\,\mathrm{skel}^2\,\tau & \text{if } \dim\sigma > 2 \end{cases}$$

Show that this task has no colorless immediate snapshot protocol.