Generalized Scans and Tri-Diagonal Systems

Paul F. Fischer Franco P. Preparata John E. Savage

Abstract

Motivated by the analysis of known parallel techniques for the solution of linear tridiagonal system, we introduce **generalized scans**, a class of recursively defined length-preserving, sequence-to-sequence transformations that generalize the well-known prefix computations (scans). Generalized scan functions are described in terms of three algorithmic phases, the reduction phase that saves data for the third or expansion phase and prepares data for the second phase which is a recursive invocation of the same function on one fewer variable. Both the reduction and expansion phases operate on bounded number of variables, a key feature for their parallelization. Generalized scans enjoy a property, called here protoassociativity, that gives rise to ordinary associativity when generalized scans are specialized to ordinary scans. We show that the solution of positive definite block tridiagonal linear systems can be cast as a generalized scan, thereby shedding light on the underlying structure enabling known parallelization schemes for this problem. We also describe a variety of parallel algorithms including some that are well known for tridiagonal systems and some that are much better suited to distributed computation.