Robust plane sweep for intersecting segments

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Abstract

In this paper, we reexamine in the framework of robust computation the Bentley-Ottmann algorithm for reporting intersecting pairs of segments in the plane. This algorithm has been reported as being very sensitive to numerical errors. Indeed, a simple analysis reveals that it involves predicates of degree 5, presumably never evaluated exactly in most implementation. Within the exact-computation paradigm we introduce two models of computation aimed at replacing the conventional model of real-number arithmetic. The first model (predicate arithmetic) assumes the exact evaluation of the signs of algebraic expressions of some degree, and the second model (exact arithmetic) assumes the exact computation of the value of such (bounded-degree) expressions. We identify the characteristic geometric property enabling the correct report of all intersections by plane sweeps. Verification of this property involves only predicates of (optimal) degree 2, but its straightforward implementation appears highly inefficient. We then present algorithmic variants that have low degree under these models and achieve the same performance as the original Bentley-Ottmann algorithm. The technique is applicable to a more general case of curved segments.