# Preferential Targeting of HIV Infected Hubs in a Scale-free Sexual Network

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**Abstract.** The structure of sexual networks plays a role in the spread of HIV. If the structure of a sexual network can be identified and the highly susceptible people (hubs) targeted, then public health intervention methods such as adult male circumcision and antiretroviral therapy can be preferentially allocated. This could better utilize HIV prevention and treatment funds as well as prevent further spread of the disease. Through the use of an epidemic simulation and assuming a scale-free sexual network we show that preferential targeting significantly decreases new HIV infections over a period of years when compared to uniform targeting. In particular, targeting the hubs in a network with antiretroviral therapy resulted in the lowest number of new infections.

**Keywords:** HIV/AIDS, scale-free network, antiretroviral therapy, circumcision

## 1 Introduction

The World Health Organisation estimated that approximately 34 million people have HIV globally as of 2011. The South African government spends millions of rand each year on HIV education, treatment and prevention programs. Despite this, South Africa has one of the highest numbers of HIV infected people in the world with an estimated 5.1 million adults infected [15]. 18% of South Africans are estimated to be infected with HIV in the high-risk age group (15-49) [15]. The numbers of infected people are increasing each year with HIV being considered a pandemic on the African continent. Prevention of HIV infection is a key strategy to control the disease. HIV behaves differently than other common viruses such as influenza. HIV has a low transmission risk on a once-off contact but it takes several years for symptoms to appear. The most common mode of transmission is through sexual contact [13]. It was concluded with a 95% confidence level that female-to-male transmission probability is 0.38% per act and male-to-female is 0.30% per act [6].

Many public health intervention methods have been undertaken to prevent HIV acquisition which are targeted to the population as a whole. Antiretroviral therapy lowers the probability of an infected person passing HIV to others. Male circumcision also lowers the chances of an uninfected male acquiring HIV. We implemented a simulator to test the effects of these intervention methods on a sexual network. The structure of the sexual network is assumed to be scale-free which means that hubs exist in the network. By targeting the intervention methods to the hubs we aim to test whether preferential targeting results in a lower number of new HIV infections.

### 1.1 Antiretroviral Therapy and Adult Male Circumcision

Antiretroviral therapy (ART) is a treatment that suppresses the HIV virus and can halt disease progression. Several factors influence the eligibility of treatment, including the CD4 cell count threshold [5]. ART has been supplied by the South African government to eligible infected people throughout the country but it is costly. Thus only a proportion of eligible people can be supplied with the treatment. Medical health insurance covers the treatment, yet millions of people in South Africa do not have access to insurance plans. The World Health Organization estimated in 2010 that 1.4 million people were receiving ART in South Africa, while 2.5 million eligible people needed ART treatment. This results in 35.9% of eligible HIV positive people receiving treatment. A study conducted by the HIV Prevention Trials Network concluded that early treatment with ART reduced HIV transmissions in couples by 96% [7].

Male circumcision is surgical removal of the foreskin, which is highly susceptible to HIV infections. In South Africa the isiXhosa circumcise as part of a coming of age initiation rite. It is also a common practice in Jewish and Islamic religions. The World Health Organization has estimated that 36% of males over the age of 15 in South Africa are circumcised [14]. Clinical trials were conducted in Kenya [9] and Uganda [8] to determine whether circumcision of adult males reduces their risk of HIV infection. In Kenya circumcised men had a 60% reduction in risk for HIV infection compared with those who were not circumcised, while the results in Uganda were a 55% reduction. The clinical trial in Uganda [8] found no evidence of reduced HIV transmission to female partners. Researchers in Johannesburg conducted a similar study and results showed that circumcised men were 60% less likely to become infected with HIV when having sex with infected women [1]. A conference held by the Joint United Nations Program on HIV/AIDS and the World Health Organization concluded that male circumcision should be a priority prevention technique due to its effectiveness in reducing men's risk of acquiring HIV. Free, incentivised adult male circumcision initiatives have been established in South Africa.

### 1.2 Scale-free Networks

In order to understand the spread of epidemics in a social network, it is crucial to understand the topology of the network. A network is scale-free if its degree distribution follows a power law. A power law does not have a peak, as a bell curve does, but is instead described by a continuously decreasing function [12].

A random network shows a bell curve degree distribution. A scale-free network has degree distribution $P(k) = ck^{-\gamma}$, where $c$ is a normalization constant and $\gamma$ is typically between two and three [2]. This means that there are few nodes in a network that have significantly more links than the majority of nodes. These nodes are referred to as hubs. The elimination of nodes that are not hubs will not disrupt the network topology significantly because they contain few links compared with the hubs. However, scale-free networks are vulnerable to coordinated attacks. When the nodes with the highest number of edges are targeted, the network breaks down faster than in the case of random node removal [2].

Several studies have been undertaken to determine whether a sexual network is scale-free. One such study took place in Sweden where data was gathered in a 1996 survey of sexual behaviour [11]. The findings of the survey show that the sexual network of questioned respondents is scale-free where $\gamma = 2.4$ for males and females. A representative sample was collected of the degree distribution of the network of sexual contacts in Ouagadougou, the capital of Burkina Faso [10]. It was found that the number of different sexual partners reported was a power law distribution with $\gamma = 2.9$. These are consistent with the degree distribution of scale-free networks.

### 1.3 Barabasi-Albert Algorithm

The origin of the power-law degree distribution observed in networks was first addressed by Barabasi and Albert [3]. Many real networks exhibit preferential attachment, such that the probability of a new node linking with an existing node is proportional to its connectivity. When choosing the nodes to which a new node in a network connects, it is assumed that the probability $p(k_i)$ that a new node will be connected to node $i$ depends on the degree $k_i$ of node $i$. The Barabasi-Albert model assumes that the probability $p(k_i)$ that a node attaches to node $i$ is proportional to the degree $k_i$ of node $i$. Thus $p(k_i)$ depends on $k_i$, in contrast to random graphs where each of the $n$ nodes is connected with independent probability $p$. The scale-free network method presented by Barabasi and Albert in 2002 is an algorithm for generating scale-free networks [4].

- Start with a small number of nodes, $m_0$.
- At each time step $s$ add a new node with linkage rate m ($\leq m_0$) links made preferentially to existing network nodes. The probability of linking to node $i$ is
$$p(k_i) = \frac{k_i}{\sum_{j=m_0+s} k_j}.$$
- Stop when the network has reached the required size N = $m_0$ + s.

## 2 Methodology

### 2.1 Assumptions

We assume that the sexual interactions of a group of people are scale-free. This is justified by the findings in Europe [11] and Africa [10]. The network represents the HIV high-risk age group of 15-49. Nodes are static and cannot leave the network (we do not take into account birth or death). 50% of the nodes are male and 50% are female. Sexual interactions are heterosexual only. ARV treatment does not take into account stage of the virus by CD4 count and is only assigned to nodes on the basis of being infected.

### 2.2 Initial Scale-Free Network and Daily Sexual Encounters

The initial sexual interaction network is built using preferential attachment from the Barabasi-Albert algorithm [4], which can then be used to run experiments. In each network generated, there are 1000 nodes, half of which are male and half are female. This network is static and displays the number of sexual partners of each node over a period of three years. Edges are formed between male and female nodes. Once the scale-free network is generated, 18% (180) of nodes are assigned as infected. It has been estimated that 62.3% of HIV infections in South Africa are female, thus 112 of the nodes assigned as infected will be female, while 68 will be male. This approximately represents people living with HIV in South Africa in the high risk group (15 - 49) [15]. The HIV assignment is random and has no relationship to the degree of the chosen nodes. Figure 1 displays a generated scale-free sexual network. The pink nodes represent females and the blue nodes represent males. The right image shows the initial allocation of 180 infected nodes by colouring infected nodes red. The average degree distribution over all generated networks is shown in Figure 2. The average degree distribution of all networks generated follows the power law required for scale-free networks where $\gamma$ is between two and three [2].

Once the network that represents all sexual interactions over a period of three years is generated, the daily sexual encounters in the network are generated. This is done by generating a new scale-free graph for each day based on the inital network created. The total degree of each node is used when calculating linkage in the Barabasi-Albert algorithm. That is, for each day a miniature version of the original network is generated using the Barabasi-Albert algorithm. The highly connected nodes are thus more likely to be chosen on a given day. This also allows concurrent partnerships to take place on a given day.

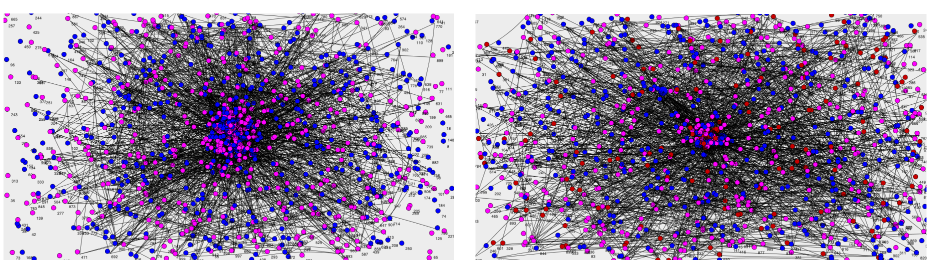

**Fig. 1.** Initial scale-free sexual network with and without HIV allocation.

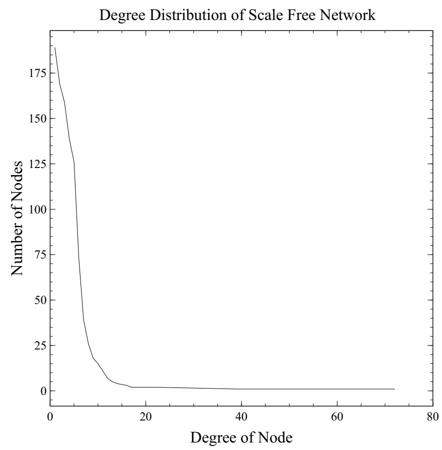


**Fig. 2.** Averaged degree distribution of generated networks.

### 2.3 Simulation

We implemented a simulator to run experiments on the sexual interaction networks, as well as provide an interface to visualize the transmission of HIV over a period of time. In the epidemic simulation a population is given with a fixed amount of nodes. The edges between all nodes within the population are known. The network's inital conditions describe how the disease starts, with which people, and the characteristics of each person with respect to the disease, which describe how likely the person will be infected. Public health interventions change the infectivity of the involved people. One interaction can take place between two nodes per time period at most. Nodes may have degree greater than one per time period to represent concurrent partners. Each discrete time period in the simulator represents one day (i.e. each iteration of the simulator). Screenshots of the simulation are displayed in Figure 3.

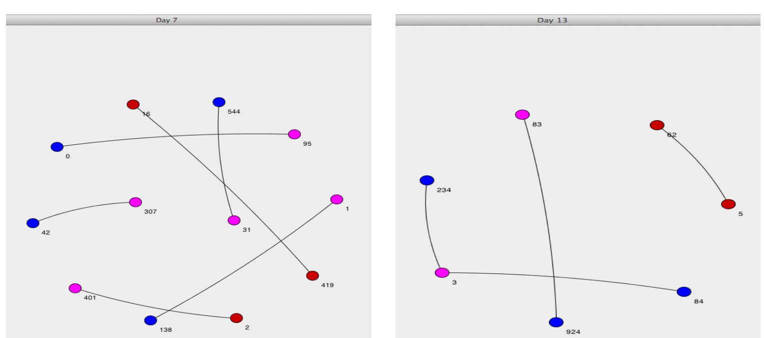


**Fig. 3.** Sexual interactions from simulation on day 7 and day 13.

### 2.4 Experimentation

Experiments are undertaken to test whether preferentially targeting highly connected nodes in a scale-free sexual network with respect to male circumcision and ART allocation can reduce the number of new HIV infections. This is illustrated and tested with the use of a simulator. During simulation, parameters are modified to reflect the probability of acquiring HIV or passing on HIV. Probabilities are changed according to public health intervention methods. These are ART allocation (which lowers the probability of passing HIV on to a healthy node), and male circumcision (which lowers the probability of a healthy male node acquiring HIV). Table 1 displays the probability per act of a healthy male node acquiring HIV depending on the condition of the female node that it shares an edge with. Table 2 displays the probabilities for a female node. These are used when running the simulation to determine the spread of HIV over three years.

**Table 1.** Probability of a healthy male acquiring HIV.

| Condition | Healthy Male |
|---|---|
| Healthy female | 0.00% |
| Infected female | 0.38% |
| Infected female on ART | 0.0152% |

**Table 2.** Probability of a healthy female acquiring HIV.

| Condition | Healthy Female |
|---|---|
| Healthy male | 0.00% |
| Infected male | 0.30% |
| Infected male on ART | 0.012% |
| Healthy circumcised male | 0.00% |
| Infected circumcised male | 0.30% |

## 3 Results

### 3.1 Initial Comparative Experiment Results

Once a scale-free sexual network was generated, the simulator was run without ART allocation or circumcision, and unprotected sex was assumed. This served as a baseline experiment. The probability of HIV infection was derived from Table 1 and Table 2. 180 nodes were initially randomly assigned as infected of which 118 were female and 62 were male. Results showed an increase of 118.33% of infected nodes on average over a period of three years.

### 3.2 ART Allocation Experiment Results

The ART allocation in South Africa obtained from the WHO estimates that 35.9% of eligible HIV positive people receive ARV therapy. Initially, 35.9% of infected nodes were randomly assigned ARV therapy, irrespective of edge degree. This reduced the probability of transmission by the infected nodes by 96% [7]. An infected female on ART will pass HIV to a healthy male with probability 0.0152%. An infected male on ART will transmit HIV to a healthy female with probability 0.012%. ART was then preferentially assigned to 35.9% of the infected nodes with the highest edge degree. The nodes were chosen by edge degree only. The simulation was run over three years and results recorded. Table 3 displays the results of the experiments on random and preferential assignment of ART to infected nodes. Five experiments were run and the results averaged.

Random ART allocation resulted in 4.1% less infected nodes on average than the experiments without intervention. The average number of infected nodes after three years with preferential assignment was significantly lower than the results obtained from random ART assignment. Preferential ART assignment to highly connected nodes resulted in 31.3% on average less infected nodes after three years than random ART assignment, and 34.1% less infected nodes on average than a scale-free sexual network without intervention. Figure 4 contains the functions of the averaged daily number of infected nodes obtained from the five experiments. The targeted ART assignment function is increasing, yet at a more linear rate than the function of non-targeted assignment. This is because highly connected nodes are more likely to have a sexual interaction on a given day, as well as more likely to have concurrent partnerships. Thus highly connected nodes become infected early on in the experiment, and receive ART treatment early on. Therefore their future partners are less likely to contract HIV as the transmission rate is lowered by 96%.

**Table 3.** Number of infected nodes initially and after random ART and preferential ART assignment experiments.

| | | Random Targeting | | Preferential Targeting | |
|---|---|---|---|---|---|
| Experiment | Initial Infected | Final Infected | Change | Final Infected | Change |
| 1 | 180 | 380 | 111.11% | 258 | 43.33% |
| 2 | 180 | 386 | 114.44% | 289 | 60.56% |
| 3 | 180 | 350 | 94.44% | 246 | 36.67% |
| 4 | 180 | 389 | 116.11% | 254 | 41.11% |
| 5 | 180 | 382 | 112.22% | 247 | 37.22% |
| **Average** | **180** | **377** | **109.44%** | **259** | **43.89%** |

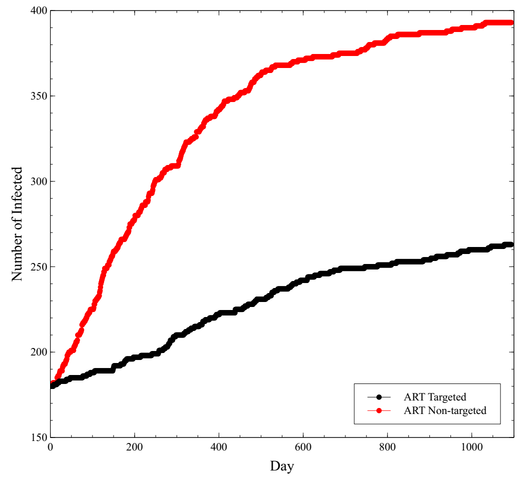


**Fig. 4.** Comparison of HIV spread through the network with random and preferential ART assignment.

### 3.3 Circumcision Experimentation Results

The scale-free network generated must reflect that 36% of the male population in South Africa are already circumcised [14]. Thus 180 male nodes were randomly assigned as circumcised, irrespective of their edge degree or health status. The probability of transmission during a once-off sexual contact from an infected female to a healthy male who is circumcised had been reduced from 0.38% to 0.152%. The results of assigning 36% of male nodes as circumcised showed that, on average, HIV infections increased from 180 to 378 after the three year period. This is a 110% increase of HIV infections on average after three years with initial circumcision allocation of 36%.

One of the HIV prevention initiatives in South Africa is to increase the number of circumcised males in the population. This is due to the lowered probability of circumcised males acquiring HIV. The initial number of nodes circumcised at the beginning of the experiment was 180. A 20% increase resulted in 100 more nodes assigned as circumcised, while a 40% increase was 200 more circumcised male nodes. For a preferential increase, nodes were ordered by their degree from highest to lowest. For a 20% targeted increase, the top 100 males nodes were chosen with respect to edge degree, bringing the total number of circumcised males to 280, of which 100 were preferentially targeted. Similarly, for a 40% increase, 200 were preferentially targeted. Table 4 displays the results of the experiments undertaken with 20% random increase and preferential increase in circumcised male nodes. Table 5 displays a 40% increase. Without intervention, 393 nodes were infected after three years on average. The average difference of infected people at the end of three years with a 20% increase in male circumcision is almost insignificant when compared to the number of infected people when keeping to the current circumcision rate of 36%.

Additional tests were run to determine how many more random male circumcisions need to take place to get the same results after three years as the targeted circumcision. For a 20% increase in circumcision, it was found that 24% of male nodes must be circumcised randomly to get the same results as when 20% of males are circumcised preferentially with respect to their high edge degree. To achieve the same HIV infection results after a three year period, 300 males must be randomly circumcised. This includes the 180 initially assigned circumcised, the 100 nodes randomly assigned circumcised by a 20% increase, and an additional 20 nodes to yield the same overall HIV infection.

**Table 4.** Infected nodes with a 20% increase in circumcision (280 circumcised).

| | | Random Targeting | | Preferential Targeting | |
|---|---|---|---|---|---|
| Experiment | Initial Infected | Final Infected | Change | Final Infected | Change |
| 1 | 180 | 385 | 113.89% | 383 | 112.78% |
| 2 | 180 | 375 | 108.33% | 371 | 106.11% |
| 3 | 180 | 365 | 102.78% | 356 | 97.78% |
| 4 | 180 | 366 | 103.33% | 362 | 101.11% |
| 5 | 180 | 382 | 112.22% | 379 | 110.56% |
| **Average** | **180** | **375** | **108.33%** | **370** | **105.56%** |

**Table 5.** Infected nodes with a 40% increase in circumcision (380 circumcised).

| | | Random Targeting | | Preferential Targeting | |
|---|---|---|---|---|---|
| Experiment | Initial Infected | Final Infected | Change | Final Infected | Change |
| 1 | 180 | 370 | 105.56% | 361 | 100.56% |
| 2 | 180 | 363 | 101.67% | 362 | 101.11% |
| 3 | 180 | 381 | 111.67% | 377 | 109.44% |
| 4 | 180 | 368 | 104.44% | 361 | 100.56% |
| 5 | 180 | 353 | 96.11% | 339 | 88.33% |
| **Average** | **180** | **367** | **103.89%** | **360** | **100%** |

## 4 Summary

Figure 6 displays the number of infected nodes over three years with and without public health intervention. All experiments begin with 180 infected nodes. The number of infected nodes is shown to increase at a decreasing rate. This is because the hubs are more likely to be chosen on a given day to have a sexual interaction and are also more likely to have more than one edge on a given

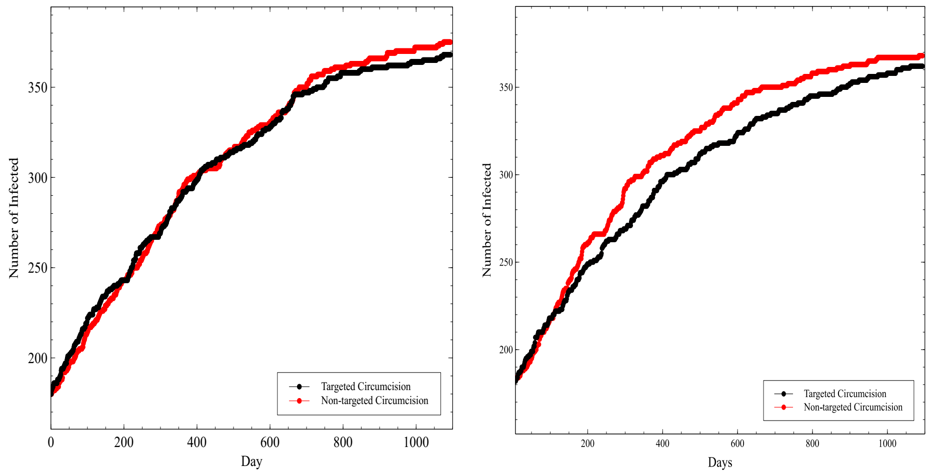


**Fig. 5.** Results of 20% and 40% increase in circumcision respectively.

day (concurrent partnerships). They are therefore more likely to contract HIV early on in the experiment. The less connected nodes are less likely to be chosen on a given day, and if chosen will likely have only one edge. Thus they are less likely to contract HIV during the simulation. The black line shows the number of infected nodes after three years assuming unprotected sex and no ART allocation or male circumcision. It displays the highest number of infected nodes after the three year period. The intervention method which results in the lowest number of overall HIV infections after three years is targeting highly connected nodes with ARV treatment.

## 5 Conclusion

Sexual networks have shown to be scale-free through empirical studies done in Africa [10] and Europe [11]. This means that the degree distribution of sexual partners within a network follow a power law. Using preferential attachment from the Barabasi-Albert algorithm, scale-free networks are generated to represent sexual networks. Experiments are done on these networks to determine the spread of HIV in a scale-free sexual network. Results showed that targeting hubs with respect to both ART and male circumcision slows the spread of HIV far more than random targeting. In experiments it is possible to perfectly identify the most highly connected nodes, however in reality this is not always possible or ethical. The most effective intervention method is targeting hubs with ART allocation. This resulted in the lowest number of HIV infections after three years. It is not realistically possible or fair to only allocate ART to highly connected nodes. If it was known that the hubs were receiving preferential treatment it could promote promiscuity. Sex workers are known hubs and they have a disproportionately higher number of sexual partners. If they can be identified a better support system could be implemented in terms of education and providing protection. However if they are found to be HIV positive, ART could be preferentially assigned to them to protect the (potentially many) future sexual partners they may encounter. The next most effective intervention method is a 40% targeted increase in male circumcision. Since this procedure is performed for free in South Africa it should be advertised and marketed to people who are more likely to be highly sexually active. As with ART targeting, it is not perfectly possible to locate these people. Thus the adult male circumcision campaigns can market their services to people who are potentially very active. An example of this is to advertise the service on condom packages. Future work on this project will remove the assumptions made. This includes taking into account homosexual relations and modelling different age groups individually.

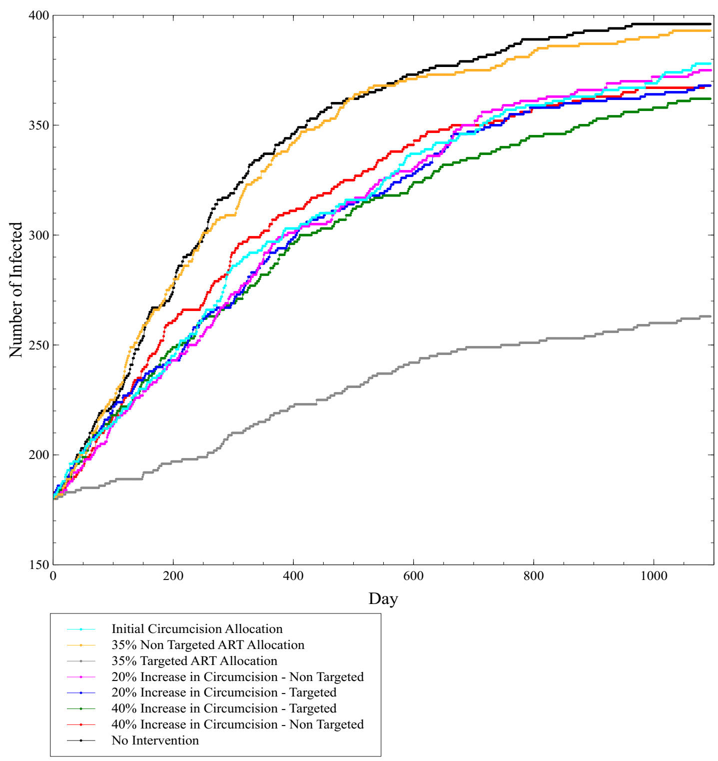


**Fig. 6.** Overall comparison of intervention methods.

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