



Paired-Dual Learning for Fast Training of Latent Variable Hinge-Loss MRFs

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Introduction

- **Latent variables** are very useful in **large-scale structured prediction**, but they **complicate learning!**
- A naive approach is not scalable, and smarter methods exist **for discrete models only**
- We introduce a **new method for continuous MRFs** that interleaves parameter and inference updates

Learning with Latent Variables

- The maximum likelihood learning objective for models with latent variables contains two inner inference problems:

$$\arg \min_w \max_{\rho \in \Delta(\mathbf{y}, \mathbf{z})} \min_{q \in \Delta(\mathbf{z})} \frac{\lambda}{2} \|\mathbf{w}\|^2 - \mathbb{E}_\rho [\mathbf{w}^\top \phi(\mathbf{x}, \mathbf{y}, \mathbf{z})] + H(\rho) + \mathbb{E}_q [\mathbf{w}^\top \phi(\mathbf{x}, \hat{\mathbf{y}}, \mathbf{z})] - H(q)$$

- At a high level, this objective has a simple structure:

Optimize \mathbf{w}

Inference in $P(\mathbf{y}, \mathbf{z} | \mathbf{x}; \mathbf{w})$

Inference in $P(\mathbf{z} | \mathbf{x}, \hat{\mathbf{y}}; \mathbf{w})$

- But repeatedly performing inference is **very expensive!**
- Supervised learning can be sped up by interleaving inference and parameter updates, e.g., Taskar et al. [ICML 2005] and Meshi et al. [ICML 2010]
- Schwing et al. [ICML 2012] and Chen et al. [ICML 2015] propose interleaving updates for discrete latent models
- Any new method for continuous variables must solve the problems of **intractable expectations and entropies**

Paired-Dual Learning

- We choose point distributions for variational families and construct entropy surrogates to make objective tractable
- Paired-dual learning replaces both inner inferences with augmented Lagrangians and optimizes jointly:

$$\arg \min_w \max_{\mathbf{y}, \mathbf{z}, \bar{\mathbf{y}}, \bar{\mathbf{z}}} \min_{\alpha} \min_{\mathbf{z}', \bar{\mathbf{z}}'} \max_{\alpha'} \frac{\lambda}{2} \|\mathbf{w}\|^2 - L_w(\mathbf{y}, \mathbf{z}, \alpha, \bar{\mathbf{y}}, \bar{\mathbf{z}}) + L'_w(\mathbf{z}', \alpha', \bar{\mathbf{z}}')$$

Optimize \mathbf{w}

Optimize $L_w(\mathbf{y}, \mathbf{z}, \alpha, \bar{\mathbf{y}}, \bar{\mathbf{z}})$

Optimize $L'_w(\mathbf{z}', \alpha', \bar{\mathbf{z}}')$

Step 1

Perform N updates over dual problem for joint distribution

Step 2

Perform N updates over dual problem for conditional distribution

Step 3

Use current state of both dual problems to compute gradient with respect to \mathbf{w} and take step in opposite direction

Hinge-Loss Markov Random Fields

- Undirected graphical models over continuous variables with hinge-loss potential functions

$$P(\mathbf{y}, \mathbf{z} | \mathbf{x}; \mathbf{w}) \propto \exp \left(\sum_{j=1}^m w_j (\max \{ \ell_j(\mathbf{x}, \mathbf{y}, \mathbf{z}), 0 \})^{p_j} \right)$$

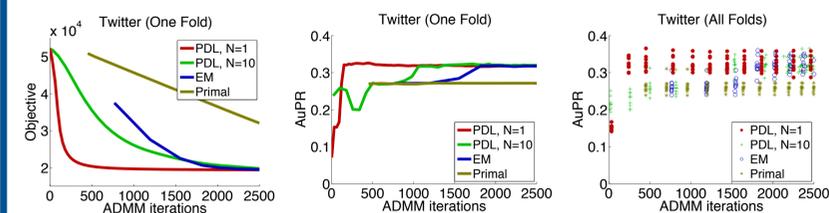
where ℓ_j is a linear function and $p_j \in \{1, 2\}$

- Generalizes
 - Randomized algorithms for MAX SAT
 - Relaxed MAP for discrete, logical MRFs
 - Exact MAX SAT using soft logic
- **Highly scalable**, ADMM-based MAP inference

Evaluation

- Paired-dual learning is
 - **Just as accurate** as traditional methods
 - **So fast that it often converges before others make a single parameter update**

Interaction Prediction via Community Detection



Trust Prediction in Social Networks

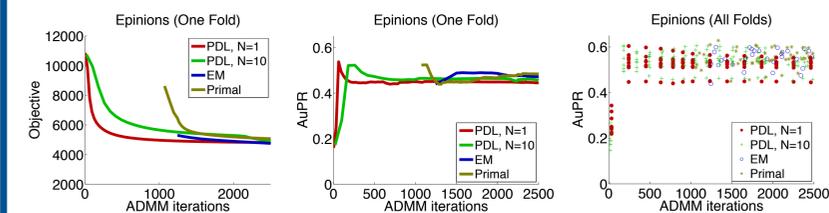
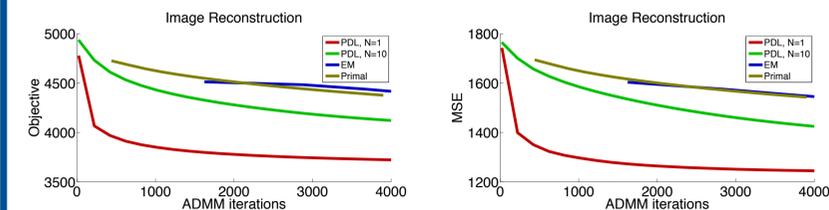


Image Reconstruction



Conclusions

- Paired-dual learning overcomes the inference bottleneck associated with learning with latent variables
- Latent variable hinge-loss MRFs are now **practical for large-scale applications!**
- Paired-dual learning is also applicable to other continuous models and even discrete models

Code available: <http://psl.cs.umd.edu>