Comparison of Bidding Algorithms for Simultaneous Auctions

Seong Jae Lee

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Brown University

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Chair

Contents

Abstract

Simultaneous auctions raise a challenge to bidders especially when there are substitutable or complementary goods. This thesis compares two classes of algorithms: marginal value-based algorithms and sampled average approximation-based algorithms, both of which are heuristics that optimize given a model of clearing prices. In the Trading Agent Competition, an annual event designed to promote research on trading agents, both algorithms are used for top players and showed almost even scores. Here, we show that sampled average approximation-based algorithms performs similar to or better than marginal value-based algorithms in both decision-theoretic setting and game-theoretic setting, especially when there is a high variance in the clearing price distribution. We claim this implies sampled average approximation-based algorithms are more adequate in a game-theoretic setting.

Thesis Supervisor: Amy Greenwald

Title: Assitant Professor of Computer Science

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I would also like to thank professor Amy Greenwald. I found this research topic from her artificial intelligence class two years ago. I still remember the day : building a TAC agent was the final project of her class, and our agent, atTACker, won the last place by a mistake. But Amy allowed me to work with her during the summer, and I have been working with her for two years since then. Without her, I could not have won at the Trading Agent Competition, published a paper, presented my work at a conference, or written this thesis. She has been a great advisor and mentor to me.

Thanks to Victor Naroditskiy, who has been working with me for two years. Fruitful discussions and encouraging support helped me to become a real researcher. Also, thanks to professor Meinolf Sellmann, who willingly agreed to be my thesis committee. Thanks to Korea Science and Engineering Foundation who gave me this great opportunity to study at Brown University. And last, but not least, thanks to my parents whom I love most.

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1 Introduction

Simultaneous auctions raise challenges to bidders, especially when there are substitutable or complementary goods on sale. Susbstitutable goods are the ones with subadditive values. For example, when one is purchasing a travel package to Miami, making a hotel reservation on Hilton and one on Raddison on the same day would not increase the value of his package. In this case, a Hilton and a Radisson are substitutable goods. On the contrary, Complementary goods are the ones with superadditive values. For example, a round-trip flight ticket and a hotel reservation are complementary goods: a travel package with no flight tickets and a Radisson has no value.

The Trading Agent Competition, TAC, is an annual event to promote researches on bidding agents. In the TAC, eight players play against one other, and each player represents a travel agency. The goal of each player is to make profit by creating travel packages and selling them to assigned eight clients with different preferences. A travel package is composed of three kinds of goods: flight tickets, hotels, and entertainment ticekts, and each of the kind is purchased through different types of auctions. For example, hotel auctions close in a random order at each minute, and their clearing prices are decided to the sixteenth highest bidding prices. On the contrary, a flight ticket is sold through a continuous auction, and its clearing price is defined as a stochastic function of time.

In this thesis, I compare two classes of heuristics : sampled average approximation (SAA)-based algorithms and marginal value (MV)-based algorithms. Both of them showed a good performance in the TAC. Table 1 shows the successful agents in the TAC using SAA-based algorithms or MV-based algorithms.

Agent	Performance	Algorithm
ATTac [SLSK01]	2000 1st, 2003 1st	MV
Walverine [CLL$^+$04]	2003 final, 2004 2nd, 2005 3rd, 2006 2nd	MV
RoxyBot [GB01]	2000 2nd, 2003 final, 2004 final	MV
RoxyBot [LGN07]	2005 final, 2006 1st	SAA

Table 1: Performance of SAA-based and MV-based algorithms in the TAC

This paper is orginized in four sections. In section 2, I explain SAA-based algorithms and MV-based algorithms. In section 3, I introduce three experiments: decision-theoretic, decision-theoretic with a noise, and game-theoretic. Each subsection contains experimental settings, results, and discussions. In section 4, we draw a final conclusion.

2 Bidding Algorithms

2.1 Sample Average Approximation

The sample average approximation (SAA) method solves stochastic optimization problems with the aid of Monte Carlo simulation [VAK+03]. The expected objective function of a problem is approximated by sampled estimates derived from a random sample. Here, we draw S number of samples from a clearing price model and search the best bidding policy $b : X \to \mathbb{R}$ that maximizes:

$$\frac{1}{|S|}\sum_{s \in S} v(Y) - s(Y) \tag{1}$$

given a set of goods X, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, and $Y := \{y \in X | s(y) < b(y)\}$.

However, there are infinite number of solutions for this equation. For example, if there is only one gods with possible clearing price 100, and if the utility of obtaining the goods is 1000, then bidding any number between 100 and 1000 is optimal. In this thesis, we developed two classes of algorithm, SAABottom and SAATop : SAABottom bids as low as possible and SAATop bids as high as possible. In case of SAATop, we set a constraint that determines the maximum limit of bidding price. The algorithms are described in the following section.

SAA-based algorithms are optimal when (i) the set of drawn samples is same as the clearing price model, and (ii) the clearing price model is the actual clearing price distribution. Unfortunately, it is impossible to fulfill these two conditions in most cases. The first condition does not hold if one does not count all the possible clearing prices, and it is even impossible if the model is continuous. Moreover, the second condition cannot be fulfilled in game-theoretic setting where the player does not know other players' strategies.

SAABottom suffers when the highest bid it considers submitting is below the clearing price. Similarly, SAATop suffers when the clearing price is higher than the highest price it expects. Because a SAA algorithm draws a finite number of samples from its distribution model, there is a possibility that it misses a clearing price within its samples. The possibility that all of the samples are smaller than the clearing price is the following :

$$\int_{-\bar{\infty}}^{\bar{\infty}} n(F(\bar{x}))^{n-1} f(\bar{x})(1 - G(x))d\bar{x} \tag{2}$$

while f is the probability mass distribution of prediction, F is the cumulative distribution function of prediction, and G is the cumulative distribution function of the clearing prices.

In a single unit auction with perfect prediction, this probability is $1/(n+1)$.

$$\begin{aligned}
&\int_{-\infty}^{\infty} n(F(x))^{n-1} f(x)(1 - F(x))dx \\
&= n\int_{-\infty}^{\infty} (F(x))^{n-1} f(x)dx - n\int_{-\infty}^{\infty} (F(x))^{n} f(x)dx \\
&= n\left[\frac{(F(x))^n}{n}\right]_{-\infty}^{\infty} - n\left[\frac{(F(x))^{n+1}}{n+1}\right]_{-\infty}^{\infty} \\
&= \frac{1}{n+1}
\end{aligned} \tag{3}$$

Therefore, when SAABottom with n scenarios bids its maximum, it may fail to complete its package with a high probability: the probability of losing a goods is $\frac{1}{n+1}$, and the probability of not losing a package composed of d number of goods is $1 - (1 - \frac{1}{1+n})^d$. For example, when $n = 50$ and $d = 10$, it is 18.0%.

2.2 Marginal Value Bidding

Given a set of goods X, a valuation function $v : 2^X \rightarrow \mathbb{R}$, and a pricing function $s : 2^X \rightarrow \mathbb{R}$. The marginal value $\mu(x, s) = \mu(x, X, v, s)$ of good $x \in X$ is defined as follows:

$$\mu(x) = \max_{Y \subseteq X \setminus \{x\}} [v(Y \cup \{x\}) - s(Y)] - \max_{Y \subseteq X \setminus \{x\}} [v(Y) - s(Y)] \tag{4}$$

In other words, the marginal value of a good is the additional value derived from owning the good relative to the set of goods one can buy.

Greenwald showed that in a decision-theoretic setting, i) $\mu(x) > s(\{x\})$, when x is within all optimal bidding sets, ii) $\mu(x) = s(\{x\})$, when x is within some optimal bidding sets, and iii) $\mu(x) < s(\{x\})$, when x is not in any of the optimal bidding sets [GN06]. Therefore, bidding marginal values on every goods in an optimal set is optimal assuming that there is only one clearing price. This is exactly what TargetMU algorithm does.

In this thesis, I used MV-based algorithms which performed well in the TAC and has been subject of many experiments [GN06] [OS06] [GB01]. TargetMU and TargetMU* are the algorithms of RoxyBot in the TAC 2000, BidEvaluator and BidEvaluator* are the algorithms of RoxyBot in the TAC 2002, and AverageMU and SampledMU are from the algorithm of ATTac. The algorithms are described in the following section.

2.3 Algorithm Descriptions

Here, I describe the algorithms used in this thesis. The number of samples from the clearing price model, or pricing functions, is determined so that each algorithm can play in the original TAC game: in the TAC, one should make a decision within a minute.

Given a set of goods X, a pricing function $s : 2^X \rightarrow \mathbb{R}$, a value function $v : 2^X \rightarrow \mathbb{R}$, the bidding function $b_{\mathsf{SAABottom}} : X \rightarrow \mathbb{R}$ with a set of pricing functions S is defined as :

$$b_{\mathsf{SAABottom}} = \mathrm{argmax}_b \frac{1}{|S|} \sum_{s \in S} (v(Y) - s(Y)) - \epsilon \sum_{y \in Y} b(y) \tag{5}$$

while $Y := \{y \in X | s(y) < b(y)\}$. Here, $|S| = 50$.

Given a set of goods X, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{SAATop}} : X \to \mathbb{R}$ with a set of pricing functions S is defined as :

$$b_{\mathsf{SAATop}} = \text{argmax}_b \frac{1}{|S|} \sum_{s \in S} (v(Y) - s(Y)) + \epsilon \sum_{y \in Y} b(y),\ b(x) < c(x)\ \forall x \in X \tag{6}$$

while $Y := \{y \in X | s(y) < b(y)\}$. Here, $|S| = 50$ and $c(x) = \max(\max_S s(x), 350 + hb(x))\ \forall x \in X$, while $hb(x)$ is the maximum hotel bonus of that good across all its clients. $350 + hb(x)$ is the maximum profit one can get when $s(x) = 0$.

Given a set of goods X, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{TargetMU}} : X \to \mathbb{R}$ with a set of pricing functions S is defined as :

$$b_{\mathsf{TargetMU}}(x) = \mu(x, \frac{1}{S} \sum_{s \in S} s)\ \forall x \in A^* \tag{7}$$

while $A^* = \text{argmax}_{Y \subset X}(v(Y) - s(Y))$, and $Y := \{y \in X | s(y) < b(y)\}$. Here, $|S| = 50$.

Given a set of goods X, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{TargetMU*}} : X \to \mathbb{R}$ with a set of pricing function S is defined as :

$$b_{\mathsf{TargetMU*}}(x) = \mu(x, s^*)\ \forall x \in A^* \tag{8}$$

while $A^* = \text{argmax}_{Y \subset X}(v(Y) - s(Y))$, $s^*(x) = \frac{1}{S} \sum_{s \in S} s(x)\ \forall x \in A^*$, and $s^*(x) = \infty\ \forall x \notin A^*$. Here, $|S| = 50$.

Given a set of goods X, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{BidEvaluator}} : X \to \mathbb{R}$ with two sets of pricing function S and S' is defined as :

$$b_{\mathsf{BidEvaluator}} = \text{argmax}_{b \in b_i} \frac{1}{|S'|} \sum_{s \in S'} (v(Y) - s(Y)) \tag{9}$$

while $Y := \{y \in X | s(y) < b(y)\}$, and $b_i = b_{\mathsf{TargetMU}}$ with a set of scenario $\{s_i\}, s_i \in S$. Here, $|S| = 25$ and $|S'| = 15$.

Given a set of goods X, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{BidEvaluator*}} : X \to \mathbb{R}$ with two sets of pricing function S and S' is defined as :

$$b_{\mathsf{BidEvaluator*}} = \text{argmax}_{b \in b_i} \frac{1}{|S'|} \sum_{s \in S'} (v(Y) - s(Y)) \tag{10}$$

while $Y := \{y \in X | s(y) < b(y)\}$, and $b_i = b_{\mathsf{TargetMU*}}$ with a set of scenario $\{s_i\}, s_i \in S$. Here, $|S| = 25$ and $|S'| = 15$.

Given a set of goods X, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{AverageMU}} : X \to \mathbb{R}$ with a set of pricing function S is defined as :

$$b_{\mathsf{AverageMU}}(x) = \frac{1}{S} \sum_{s \in S} \mu(x, s) \; \forall x \in X \tag{11}$$

Here, $|S| = 15$.

Given a set of goods X, a pricing function $s : 2^X \to \mathbb{R}$, a value function $v : 2^X \to \mathbb{R}$, the bidding function $b_{\mathsf{StraightMU}} : X \to \mathbb{R}$ with a set of pricing function S is defined as :

$$b_{\mathsf{StraightMU}}(x) = \mu(x, \frac{1}{S} \sum_{s \in S} s) \; \forall x \in X \tag{12}$$

Here, $|S| = 50$.

3 Experiments

3.1 General Experimental Setting

The Trading Agent Competition is designed to promote research on trading agents. In a game, 8 players participate, and each player represents a travel agency with 8 clients. The goal of a player is to create travel packages that maximize its profit by procuring goods from different types of auctions. A travel package is composed of three kinds of goods - a round-trip flight, a hotel reservation, and entertainment tickets -, which are purchased from different kinds of auctions. The profit of a player is determined by the preferences of its clients, travel packages, and the cost of procured goods. Precisely, the profit per a client is determined as :

$$\text{profit} = 1000 - \text{travel penalty} - \text{cost} + \text{hotel bonus} + \text{fun bonus} \tag{13}$$

while travel penalty is defined as 100 times the sum of i) the difference of preferred arrival date and actual arrival date, and ii) the difference of preferred departure date and actual departure date, hotel bonus is the bonus when a client gets the nicer hotel, and fun bonus is the bonus when a client gets entertainment tickets. Hotel bonus and fun bonus is determined randomly for each client.

A travel package should be within day 1 and day 5, thus there are 4 inbound flight tickets and 4 outbound flight tickets. There are two kinds of hotel, Tampa Towers and Shoreline Shanties, and every client prefer staying at the former. Eight auctions are held for each hotel and each day. Finally, there are three kinds of entertainment tickets for each day.

Flights can be purchased continuously, and the clearing prices is defined as a stochastic function of time with a hidden parameter. Entertainment tickets can be purchased in continuous double auctions from other players - at the beginning of a game, entertainment tickets are distributed to players randomly. Finally, hotel auctions close at each minute in a random order, and each auction distributes 16 rooms of same type of hotel on same day with the 16th highest bidding price as a clearing price. The 16th highest bidding price of each auction is notified to players every minute.

We modified this game setting into a simultaneous one: all auction closes simultaneously. To make it simpler, flight ticket prices are fixed to 325 and entertainment tickets are removed.

3.2 Experiment with Perfect Prediction

3.2.1 Experimental Setting

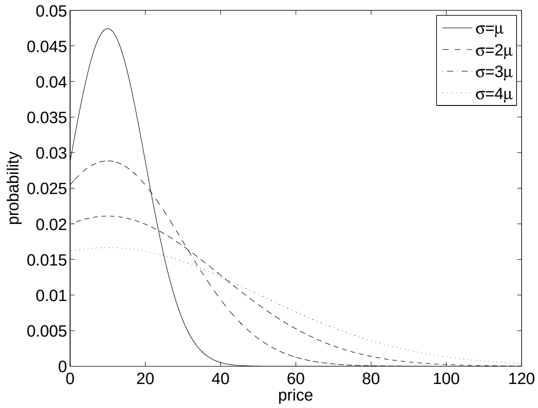
In the first experiment, prediction and clearing prices are sampled from the same normal distribution with mean $\bar{\mu} = (10, 50, 50, 10, 40, 110, 110, 40)$ and standard deviation $\bar{\sigma} = \lambda\bar{\mu}$. $\bar{\mu}$ is close to the TAC's competitive equilibrium price. Each auction's clearing price is sampled independently. When the price is sampled below zero, it is resampled. Figure 1(a) shows the probability distribution of the clearing prices of a hotel with $\mu = 10$. I omitted other distribution graphs, because the shape of distributions would be basicaly same to the shown graph, while the unit of x axis would be proportional to their μ. I ran four experiments with parameter $\lambda = \{1, 2, 3, 4\}$, 1000 games for each.

In the second experiment, prediction and clearing prices are sampled from the normal distribution with mean $\bar{\mu} = (150, 150, 150, 150, 250, 250, 250, 250)$ and standard deviation σ. Again, each auction's clearing price is sampled independently. When the sampled price is below zero, it is resampled. Figure 1(b) shows the probability distribution of the clearing prices of a hotel with $\mu = 150$. With low variance, the shape of the distribution is same for other auctions. When variance is high enough, the probability distribution would be different due to resampling. I ran eight experiments with parameter $\sigma = \{1, 20, 40, 60, 80, 100, 120, 140\}$, 1000 games for each.

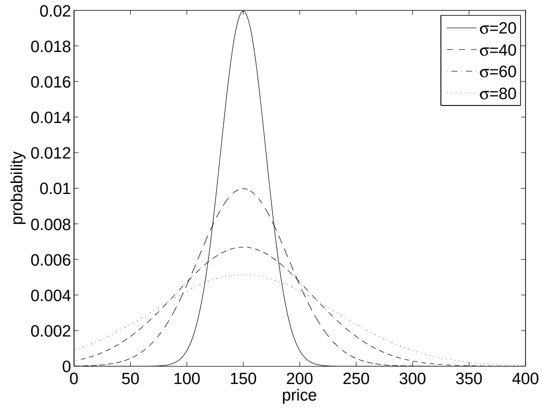
3.2.2 Result

The score mean of each algorithm is shown in Figure 2. Confidence intervals of score mean and detailed game result tables are shown at the Appendix A.

In the first experiment, SAATop, BidEvaluator*, and TargetBidder* perform best when $\sigma = 1$. As variance increases, the scores of BidEvaluator* and TargetBidder* drop faster than those of SAATop and SAABottom, making the latter top players in a high variance setting. One should note that the drastic fall of scores is caused not only by the changes on variance but also by the changes on mean: since the sampling method truncates the samples with



(a) $\bar{\mu} = (10, 50, 50, 10, 40, 110, 110, 40)$



(b) $\bar{\mu} = (150, 150, 150, 150, 250, 250, 250, 250)$

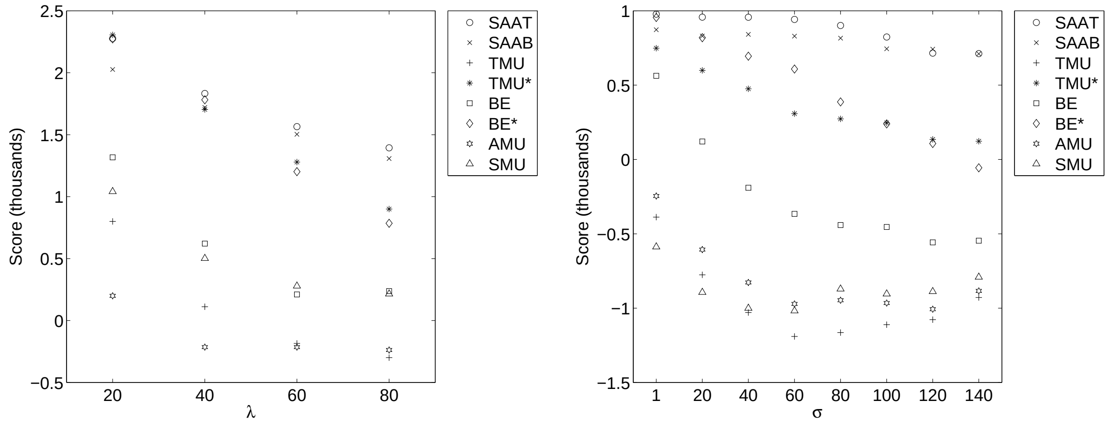
Figure 1: Clearing price distribution, experiment with perfect prediction

negative price, the mean of the model increases as its variance does. For example, when $\lambda = 4$, the actual mean of the distribution is (37, 180, 180, 39, 145, 384, 378, 145).

In the second experiment, SAATop and BidEvaluator* perform best when $\sigma = 1$. However, the scores of SAA-based algorithms drop slower than those of MV-based algorithms as variance increases. When $\sigma = 140$, the order of the algorithms is SAATop, SAABottom, TargetBidder*, BidEvaluator*, BidEvaluator, TargetBidder, AverageMU and StraightMU, starting from the highest score. This is consistent with the order of algorithms in the first experiment, $\lambda = 80$.

3.2.3 Discussion

There is a score gap between SAATop and SAABottom. With the same scenario set, they aim the same set of goods for each scenario. The only difference between them is their bidding prices: SAATop places as high as possible to win the same goods, while SAABottom places as low as possible to win those. Therefore, SAABottom is more likely to lose goods that it was sure that it would win them. When one is bidding on a set of complementary goods, losing a goods means not only losing the expected profit, but also losing all the complementary goods it purchased: in this experiment, SAABottom has to throw a pair of round trip flight tickets out.



(a) $\bar{\mu} = (10, 50, 50, 10, 40, 110, 110, 40)$ (b) $\bar{\mu} = (150, 150, 150, 150, 250, 250, 250, 250)$

Figure 2: Score mean, experiment with perfect prediction

We can calculate the number of unexpected failure of winning aimed goods for SAABottom from Equation 2. In the second experiment, SAABottom would unexpectedly fail to complete a travel package with probability $\frac{1}{51}^{d}$. Since the number of used hotels is almost a half of the number of used flights, we can assume $d = 1$ (Table 7). Also, Figure 3 shows that SAA-based algorithms bid more than 95% of goods with the maximum price when $\sigma = 20$: the right part of bidding price distribution resembles the highest sampled price distribution. Therefore, the number of unexpected failure per a travel package would be about $\frac{1}{51}$. Considering that the average number of completed travel packages for SAATop is 7.1, the number of unexpected failure per a game would be about 0.14 ($7.1 \times \frac{1}{51}$). The number of null package for SAABottom is greater than that of SAATop by 0.22. We can say a great part of the mal-performance of SAABottom came from this unexpected failure.

For the second experiment with $\sigma = 20$ (Table 7), the number of unused flight tickets per game is 0.01 for SAATop, and 0.38 for SAABottom (Table 7). SAABottom buys two flight tickets (650), while SAATop buys two flight tickets (650) and a hotel reservation (200) but compensated with utility (6997/(8-0.9)=985). Therefore, each failure on aimed hotel makes SAABottom earns 785 less than SAATop. Since the difference on the number of unused flight tickets is 0.37, we can infer SAABottom lost in hotel auctions 0.19 more per game. This gives almost 150 points off from SAABottom. From the table, we can see the score difference between two algorithms is 193.

However, in the second experiment, the performances of SAATop and SAABottom become similar as variance increases. This happens because the bidding price of SAATop and SAABottom becomes similar. Figure 3 shows that when $\sigma = 80$, the bidding price distribution of SAABottom and that of SAATop shows more similarity. Table 13 shows that the difference of the number of unused flight between SAATop and SAABottom is 0.05 with $\sigma = 140$, while it is 0.22 with $\sigma = 20$.

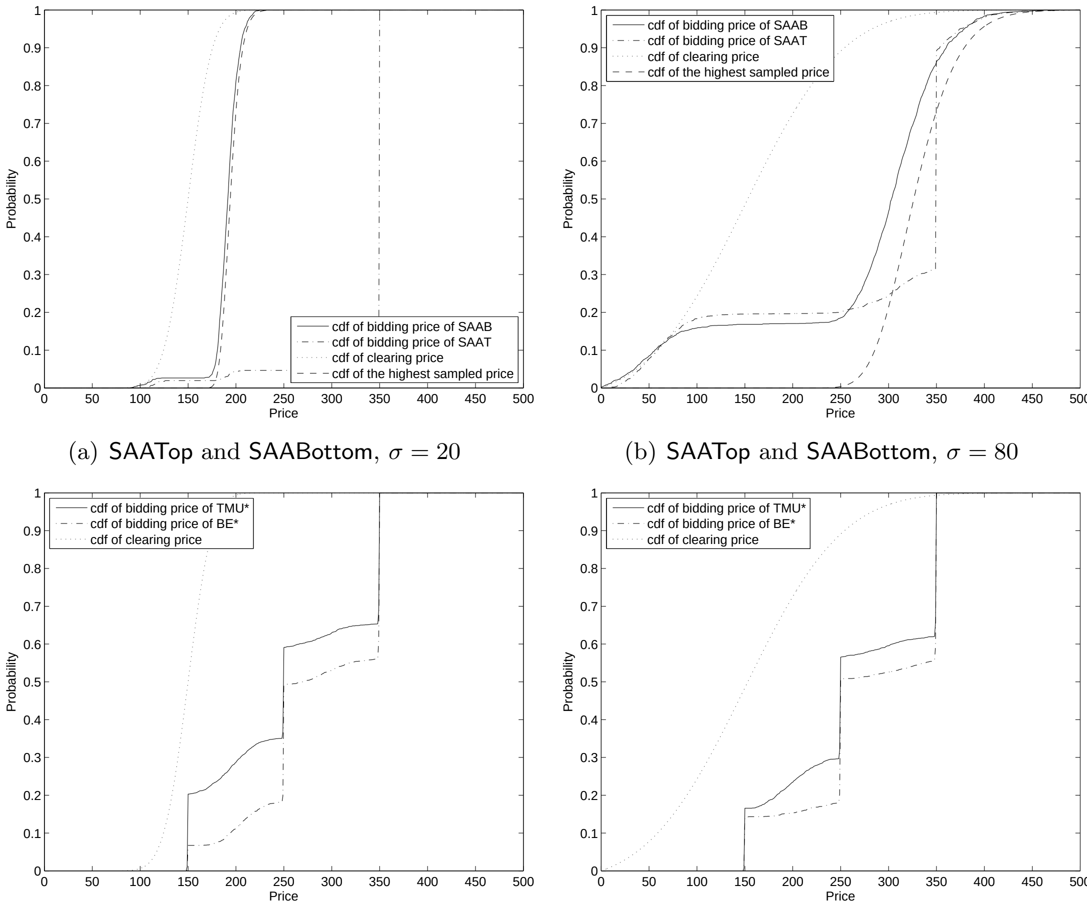
Another thing to note is that Figure 3 shows that the maximum amount SAATop bids is more than the marginal value of the good. Since the maximum possible marginal value is 350 plus a hotel bonus (utility 1000 minus two flight tickets 650 plus a hotel bonus), MV-based algorithms will never bid more than 350. On the contrary, the maximum amount that an optimal algorithm can bid is 1000.

Both experiments show that SAA-based algorithms are more tolerant to the variance of the clearing price distribution compared to MV-based algorithms. The performance of the latter decrease faster than that of the former as variance increases. Figure 3(c) and Figure 3(d) shows two best MV-based algorithms are more likely to lose goods in the higher variance setting.

3.3 Experiment with Noisy Prediction

3.3.1 Experimental Setting

In this experiment, prediction is sampled from the normal distribution with mean $\bar{\mu}$ = (150, 150, 150, 150, 250, 250, 250, 250) and the given parameter σ. The clearing price is shifted by the given parameter λ from the same distribution. For example, when $\sigma = 20, \lambda = -40$, prediction is sampled from the normal distribution with mean $\bar{\mu}$ = (150, 150, 150, 150, 250, 250, 250, 250) and $\sigma = 20$, and clearing price is sampled from the normal distribution with mean $\bar{\mu}$ = (110, 110, 110, 110, 210, 210, 210, 210) and $\sigma = 20$. I ran experiments with parameter λ = {-40, -30, -20, -10, 10, 20, 30, 40} and σ = {20, 80}, 500 games for each.



(a) SAATop and SAABottom, $\sigma = 20$

(b) SAATop and SAABottom, $\sigma = 80$

(c) TargetBidder* and BidEvaluator*, $\sigma = 20$

(d) TargetBidder* and BidEvaluator*, $\sigma = 80$

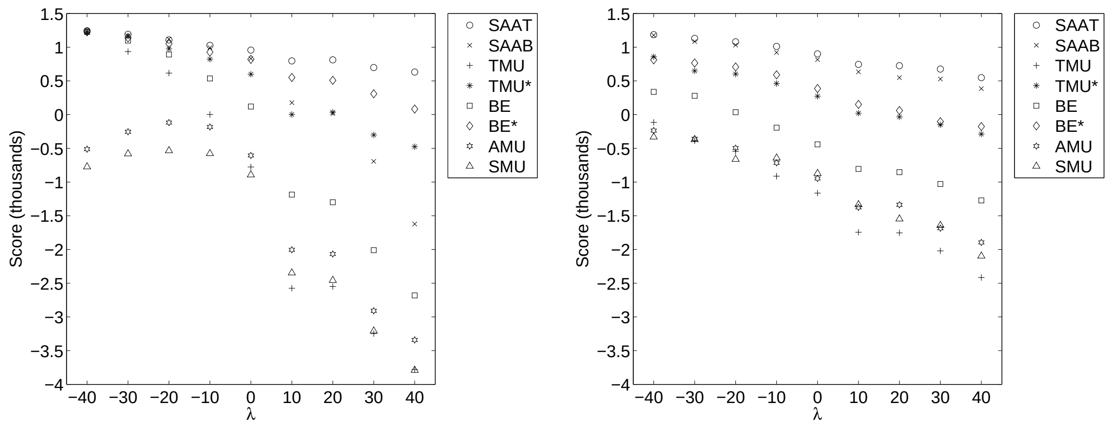
Figure 3: Cdf of bidding prices for cheap hotels, experiment with perfect prediction

3.3.2 Result

The mean score is shown in Figure 4. Confidence intervals of score mean and detailed game result tables are shown at the Appendix B.

In the first experiment with $\sigma = 20$ (Figure 4(a)), all the algorithms except AverageMU and StraightMU perform best when $\lambda = -40$, that is, when the mean of the clearing price is lower than the mean of prediction by 40. As λ increases, the order of the algorithms changes to SAATop, BidEvaluator*, TargetBidder*, SAABottom, BidEvaluator, AverageMU, StraightMU and TargetBidder from the highest score. In general, the score of an algorithm is higher when λ is negative, although there are exceptions for AverageMU and StraightMU.

In the second experiment with $\sigma = 80$ (Figure 4(b)), the order of the algorithms are consistent across all λ in the order of SAATop, SAABottom, BidEvaluator*, TargetBidder*, BidEvaluator, AverageMU, StraightMU and TargetBidder from the highest score. In general, the score of an algorithm is higher when λ is negative.



(a) $\sigma = 20$ (b) $\sigma = 80$

Figure 4: Score mean, experiment with noisy prediction

3.3.3 Discussion

In the first experiment, there is very small difference in score between all the algorithms except AverageMU and StraightMU when $\lambda = -40$. Table 14 shows that these algorithms created a single hotel package in the most of the time, because the number of used flights is twice the number of used hotels. Moreover, they bid only the hotels they wanted to complete, because the number of purchased hotels is same as the number of used hotels. With these bidding polices, the problem can be reduced to the bidding problem with a single clearing price. When $\sigma = 20$ and $\lambda = -40$, the clearing price will be lower than the mean price of scenarios most of the time, because 95% of the clearing prices would be below $\mu - 20$, and 95% of the prices in a scenario would be above. Since TargetBidder bids at least the mean price of the scenarios on aimed goods, it will hardly fail winning its bids. In a similar way, BidEvaluator will also hardly fail winning its aimed bids, since

BidEvaluator bids at least the price of a scenario. Because TargetBidder* and BidEvaluator* bids higher than TargetBidder and BidEvaluator respectively, these four algorithms - TargetBidder, TargetBidder*, BidEvaluator and BidEvaluator* - will be sub-optimal.

The drastic fall of SAABottom can also be explained in the same way. The bidding price of SAABottom is less than the maximum price of its prediction plus ϵ, because SAABottom assumes that it would win any of the bids with this price. When $\lambda = 40$ and $\sigma = 20$, the clearing price will be greater than most of the price SAABottom bids, since 95% of the clearing prices would be above $\mu + 20$, while 95% of the prices in a scenario would be below. If we assume most of the packages are single hotel packages, the probability that SAABottom unexpectedly fails to complete a travel package is 41.31% from Equation 2. Table 21 shows the number of hotels won by SAABottom is only 54.93% of that by SAATop. On the contrary, when $\lambda = 40$ and $\sigma = 80$, the ratio is 5.50%. Table 29 shows the number of hotels won by SAABottom is 94.12% of that by SAATop.

The scores of AverageMU and StraightMU increase for a while and then decrease as λ increases. increase is caused by winning their bids too much, and the decrease is due to losing their bids too much.

3.4 Experiment with CE Prediction

3.4.1 Experimental Setting

Here, I present the experiments with game-theoretic settings. More than one players place bids on each auction, and those who placed bids greater than or equal to the sixteenth highest bidding price will win the goods with the price. Because we cannot know what the clearing prices will be in a game-theoretic setting, the quality of prediction matters. There is a prediction method to calculate the competitive equilibrium (CE) price of the TAC game proposed by TAC agent Walverine [CLL+04]. Here, I used its modified algorithm, using a simultaneous ascending auction technique, which is used in RoxyBot-06 [LGN07]. This algorithm increases a price while the total demand is greater than the total supply starting from zero price (Equation 14).

$$P_{n+1} = P_n + \mathrm{MAX}(0, \alpha P_n(\mathrm{demand} - \mathrm{supply})), P_0 = 0 \tag{14}$$

It is different from the prediction used in previous experiments: it is not a normal distribution, and the price of an auction is dependent on those of other auctions. For example, the prices of hotels on the same day have a high covariance, because they are substitutable goods.

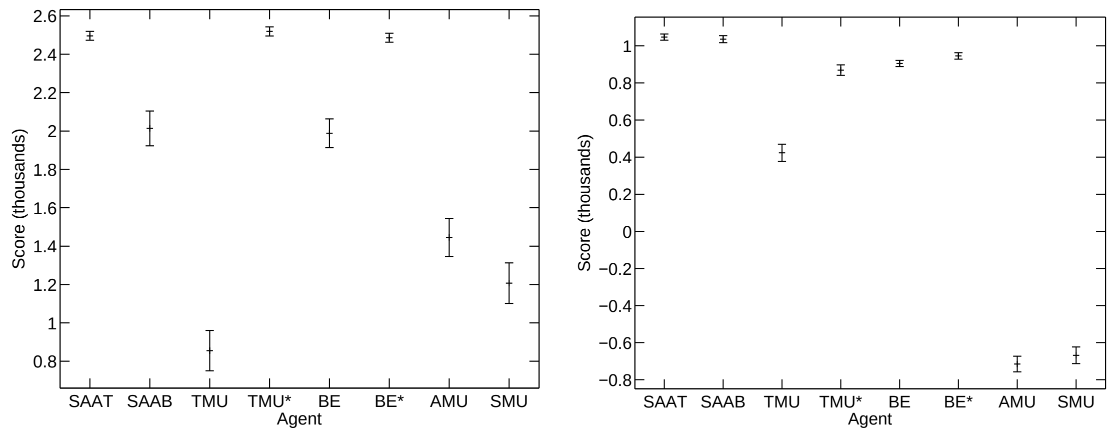
For the first experiment, the number of palyers in the game is eight as a normal TAC game does. For the second experiment, the number of players in the game follows a binomial distribution with parameter 32 and 0.5. In the second experiment, a player chooses its algorithm randomly. 1500 games were played for the first experiment, and 1046 games were played for the second experiment.

3.4.2 Result

The mean score is shown in Figure 5. Detailed game result tables are shown at the Appendix C. In the first experiment, SAATop, TargetBidder* and BidEvaluator* are the top algorithms, follwed by SAABottom and BidEvaluator. In the second experiment, SAATop and SAABottom are the top algorithms, followed by BidEvaluator*, BidEvaluator, and TargetBidder*.

Figure 6(a) and Figure 6(c) show the distribution of CE prediction and that of clearing prices for the first experiment. The mean of prediction was (2, 57, 57, 2, 30, 97, 97, 30), and its standard deviation was (7, 23, 23, 7, 24, 14, 14, 25). The mean of clearing prices was (1, 53, 53, 1, 29, 95, 96, 31), and its standard deviation was (4, 29, 29, 4, 29, 20, 20, 30).

Figure 6(b) and Figure 6(d) show the distribution of CE prediction and that of clearing prices for the second experiment. The mean of CE prediction was (137, 155, 155, 137, 239, 255, 255, 239), and its standard deviation was (55, 45, 45, 55, 60, 53, 53, 60). The mean of clearing price was (123, 127, 126, 127, 229, 219, 219, 230), and its standard deviation was (59, 52, 52, 56, 73, 71, 72, 70).



(a) 8 players (b) B(32, 25) players

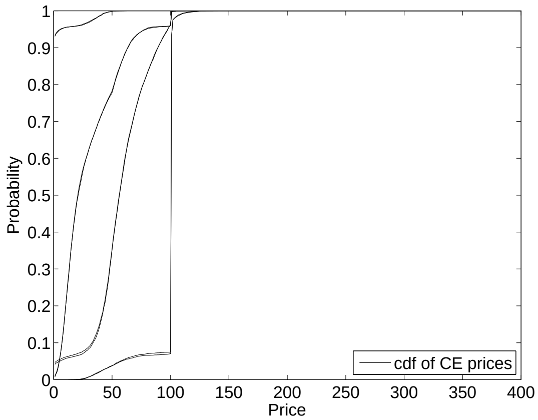
Figure 5: Confidence intervals of score mean, experiment with CE

3.4.3 Discussion

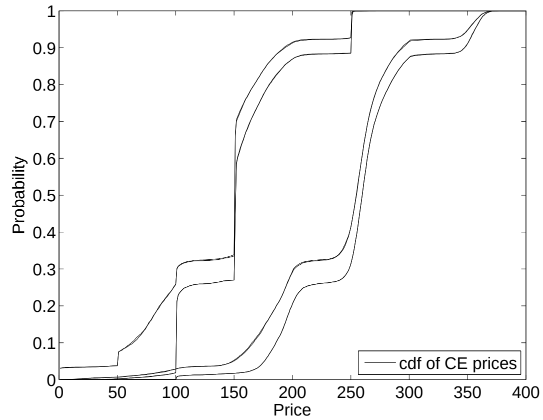
In both experiments, the mean of prediction is higher than that of the clearing prices and the standard deviations of prediction is lower than that of the clearing prices. I think the quality of the prediction is pretty good, although it is overpredicted by about 20. As we have seen from the previous experiments with over-prediction, it would just make us harder to see the order of the algorithms. Still, if there is a score difference between algorithms, the order between them will be preserved even with more accurate prediction methods.

We can compare the second experiment with the experiment with μ = (150, 150, 150, 150, 250, 250, 250, 250) and $\sigma = 60$, because its mean and standard deviation of clearing prices are similar to those of this experiment. The general order is similar, but TargetBidder and BidEvaluator is placed on a higher order in the game-theoretic setting. The second experiment also shows results similar to those of the first experiment.

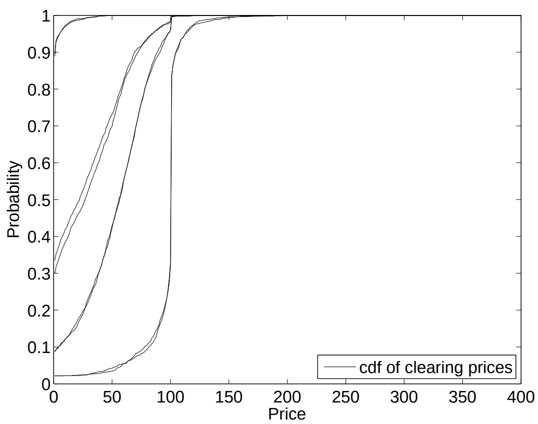
The results show that in the first experiment, it is hard to measure the performances among SAATop, BidEvaluator* and TargetBidder*. This is consistent with the fact that some MV-based algorithms performed as well as SAA-based algorithms in the TAC games. Still, as we can see in the second experiments, there is a case that MV-based algorithms performs worse than SAA-based algorithms. Note that in Figure 5(b), the performances



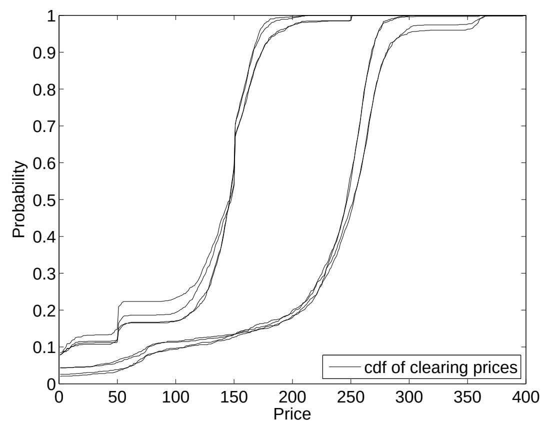
(a) Cdf of CE, 8 agents



(b) Cdf of CE, B(32,0.5) agents



(c) Cdf of clearing prices, 8 agents



(d) Cdf of clearing prices, B(32,0.5) agents

Figure 6: Cdf of CE prices and clearing prices, experiment with CE

of TargetBidder, BidEvaluator and BidEvaluator* might seem to be similar to those of SAA-based algorithms, but in reality, the formers performs strictly worse than the latter: the extremely low scores of AverageMU and StraightMU make the formers look better.

4 Conclusion

In this thesis, I tried to compare SAA-based algorithms and MV-based algorithms in a simultaneous auction. In general, SAA-based algorithms perform better than the others. First, they are optimal in a decision-theoretic setting with infinite number of scenarios. Second, in a decision-theoretic setting, they are more tolerant to variance. Third, in a decision-theoretic setting, they are more tolerant to noise, especially for SAATop. Additionaly, we found that SAABottom performs as well as SAATop in high variance settings. Finally, SAA-based algorithms showed a better performance even in the game-theoretic setting we illustrated.

However, we saw that BidEvaluator* and TargetBidder* perform as well as SAATop when there is a small variance on the distribution of clearing prices. With a small variance, the bidding problem with the clearing price distribution can be approximated to the bidding problem with a single clearing price, in which setting both algorithms are optimal.

Therefore, I suggest using a sample average approximation-based algorithm rather than using a marginal value-based algorithm for bidding problems. Although the TAC showed some MV-based algorithms perform best, there are cases that MV-based algorithms even fails to make profit. On the contrary, SAA-based algorithms perfom optimally when the prediction quality is good and the resource is enough.

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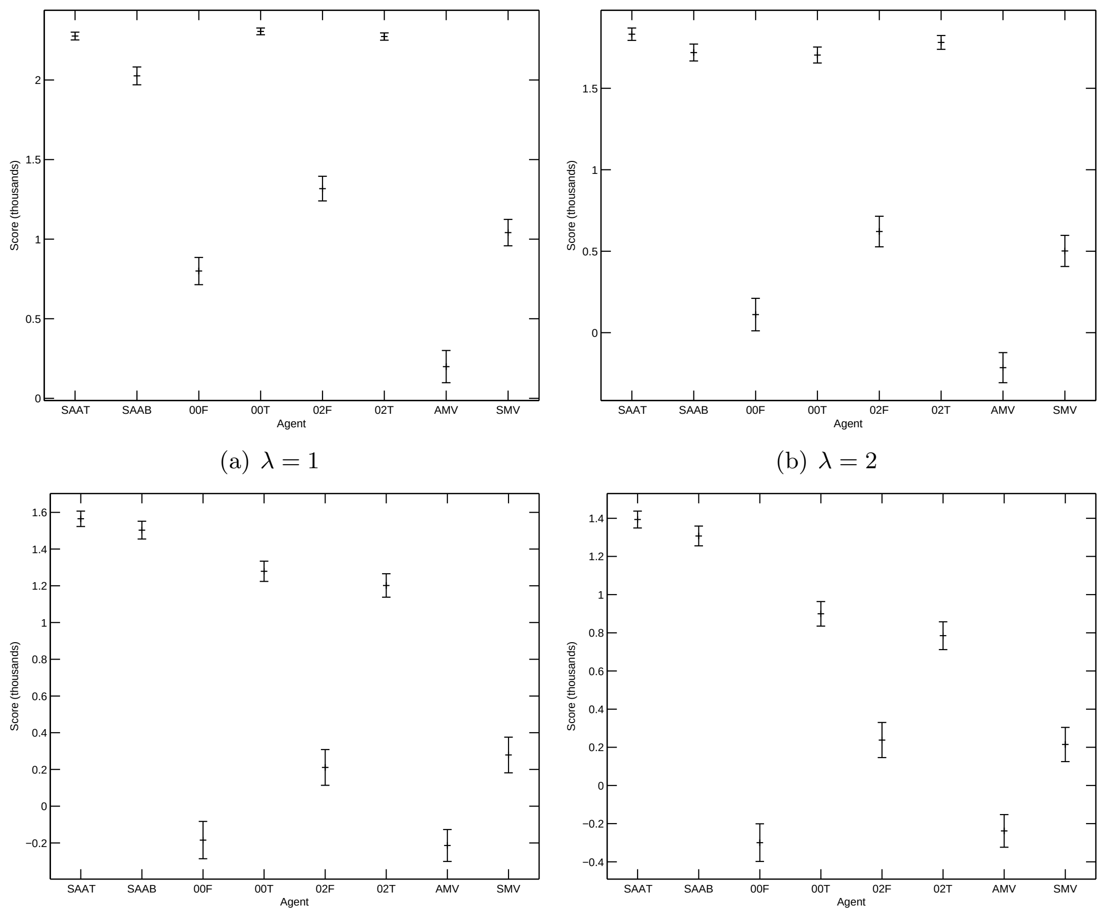
Appendix

A. Experiment with Perfect Prediction



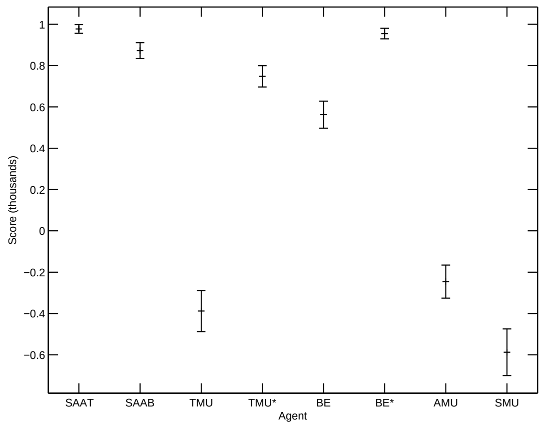
(a) $\lambda = 1$

(b) $\lambda = 2$

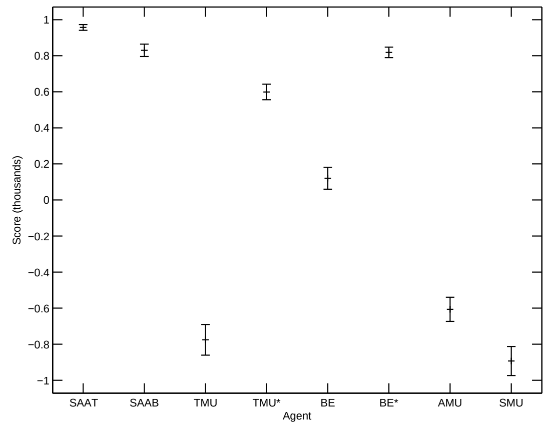
(c) $\lambda = 3$

(d) $\lambda = 4$

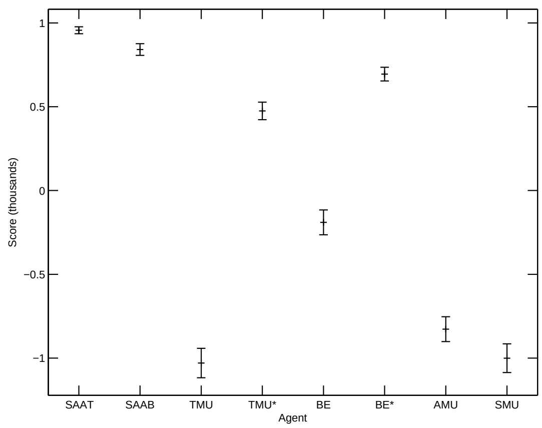
Figure 7: Confidence intervals of score mean, experiment with low mean



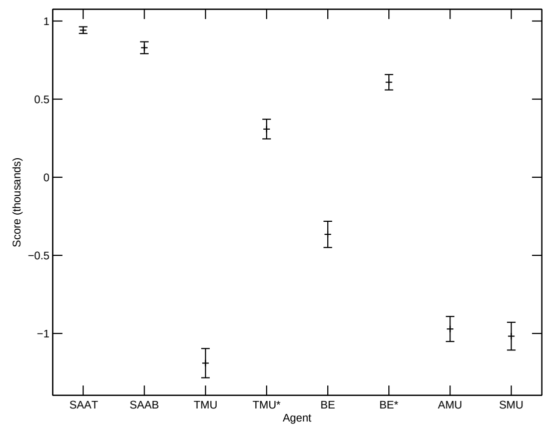
(a) $\sigma = 1$



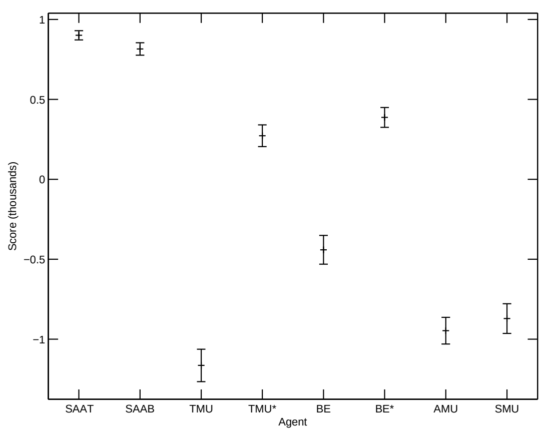
(b) $\sigma = 20$



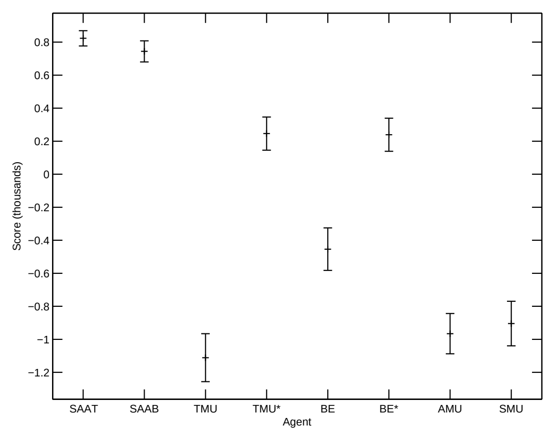
(c) $\sigma = 40$



(d) $\sigma = 60$

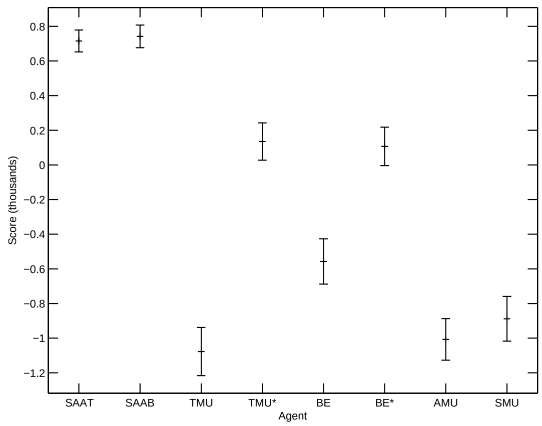


(e) $\sigma = 80$

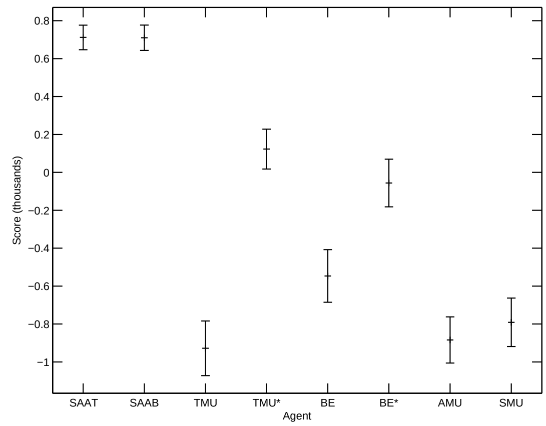


(f) $\sigma = 100$

Figure 8: Confidence intervals of score mean, experiment with high mean I



(a) $\sigma = 120$



(b) $\sigma = 140$

Figure 9: Confidence intervals of score mean, experiment with high mean II

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	2276	2026	799	2305	1317	2272	199	1041
Utility	8220	7939	6464	8265	6996	8233	5821	6849
Cost	5944	5913	5664	5959	5678	5960	5622	5807
Penalty	282	268	244	275	375	274	77	247
Null	0.04	0.31	1.73	0.01	1.09	0.02	2.48	1.38
F.unuse	0.09	0.62	3.47	0.02	2.18	0.04	4.57	2.75
F.use	15.9	15.4	12.5	16.0	13.8	16.0	11.0	13.2
F.cost	5200	5200	5200	5200	5200	5200	5072	5200
F.aver	325	325	325	325	325	325	325	325
H.bonus	547	520	443	553	460	529	384	474
H.earn	13.7	13.4	10.8	13.1	10.4	13.2	14.9	14.6
H.unuse	0.7	0.9	1.6	0.0	0.8	0.0	6.2	4.6
H.use	13.1	12.5	9.2	13.1	9.5	13.1	8.7	10.1
H.cost	744	713	464	759	478	760	550	607
H.aver	54.2	53.2	42.9	58.1	46.2	57.8	36.8	41.5

Table 2: Experiment with low mean, $\lambda = 1$

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	1832	1719	111	1704	620	1781	-214	501
Utility	7822	7652	5727	7702	6218	7852	5165	6399
Cost	5990	5933	5616	5998	5597	6071	5380	5897
Penalty	558	568	380	534	525	453	174	416
Null	0.16	0.30	2.29	0.30	1.61	0.15	3.00	1.67
F.unuse	0.32	0.61	4.59	0.60	3.21	0.30	4.97	3.35
F.use	15.7	15.4	11.4	15.4	12.8	15.7	10.0	12.7
F.cost	5200	5200	5200	5200	5200	5200	4866	5200
F.aver	325	325	325	325	325	325	325	325
H.bonus	539	526	403	537	349	455	338	490
H.earn	12.1	11.6	7.8	10.2	7.5	11.3	13.0	13.7
H.unuse	2.1	2.0	1.1	0.3	0.5	0.1	6.6	6.1
H.use	9.9	9.6	6.7	10.0	7.1	11.2	6.4	7.6
H.cost	790	733	416	798	397	871	514	697
H.aver	65.5	63.1	53.1	78.1	52.7	77.2	39.6	50.9

Table 3: Experiment with low mean, $\lambda = 2$

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	1564	1503	-184	1278	211	1201	-214	278
Utility	7489	7390	5373	7162	5755	7144	4994	6259
Cost	5924	5887	5557	5883	5544	5942	5208	5980
Penalty	779	751	461	676	613	683	255	574
Null	0.20	0.29	2.46	0.57	1.85	0.57	3.07	1.63
F.unuse	0.40	0.58	4.93	1.15	3.67	1.13	4.66	3.27
F.use	15.6	15.4	11.1	14.9	12.3	14.9	9.9	12.7
F.cost	5200	5200	5200	5200	5193	5199	4719	5201
F.aver	325	325	325	325	325	325	325	325
H.bonus	470	431	300	412	215	396	319	469
H.earn	10.7	10.3	5.8	7.7	6.5	8.5	11.5	13.4
H.unuse	2.9	2.5	0.2	0.1	0.2	0.2	6.1	6.9
H.use	7.8	7.8	5.6	7.6	6.3	8.3	5.5	6.5
H.cost	724	687	357	683	350	744	489	779
H.aver	67.7	66.7	62.0	88.8	54.1	87.5	42.4	58.2

Table 4: Experiment with low mean, $\lambda = 3$

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	1393	1307	-299	899	238	784	-238	214
Utility	7375	7244	5237	6702	5729	6504	4855	6280
Cost	5981	5937	5537	5802	5491	5719	5093	6066
Penalty	782	786	486	655	673	765	307	603
Null	0.21	0.31	2.47	0.91	1.72	0.97	3.13	1.54
F.unuse	0.42	0.61	4.94	1.81	3.32	1.82	4.36	3.09
F.use	15.6	15.4	11.1	14.2	12.6	14.1	9.7	12.9
F.cost	5200	5200	5200	5200	5160	5164	4581	5200
F.aver	325	325	325	325	325	325	325	325
H.bonus	365	339	193	264	124	235	295	427
H.earn	10.6	10.2	5.5	7.1	6.4	7.2	11.0	13.9
H.unuse	2.9	2.5	0.0	0.0	0.1	0.0	5.9	7.5
H.use	7.8	7.7	5.5	7.1	6.3	7.2	5.2	6.5
H.cost	781	737	337	602	331	555	512	866
H.aver	73.4	72.1	61.0	84.9	52.0	77.0	46.4	62.2

Table 5: Experiment with low mean, $\lambda = 4$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	977	872	-388	748	562	955	-245	-587
Utility	7139	6594	5250	6725	6105	6488	5409	5676
Cost	6162	5722	5638	5977	5542	5533	5655	6264
Penalty	564	450	254	454	352	396	254	305
Null	0.75	1.40	2.82	1.26	1.96	1.57	2.70	2.40
F.unuse	0.00	0.26	3.80	0.72	1.16	0.08	2.56	3.18
F.use	14.5	13.2	10.4	13.5	12.1	12.9	10.6	11.2
F.cost	4713	4375	4603	4616	4300	4204	4276	4675
F.aver	325	325	325	325	325	325	325	325
H.bonus	454	447	322	439	422	458	365	382
H.earn	7.3	6.6	5.2	6.7	6.0	6.4	6.9	8.0
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	1.6	2.4
H.use	7.3	6.6	5.2	6.7	6.0	6.4	5.3	5.6
H.cost	1449	1347	1035	1360	1242	1329	1380	1589
H.aver	200.0	204.3	199.8	201.9	205.8	206.9	200.1	199.5

Table 6: Experiment with high mean, $\sigma = 1$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	957	830	-775	599	120	818	-606	-893
Utility	6997	6804	4708	6472	5605	6351	4788	5413
Cost	6040	5974	5484	5872	5484	5533	5394	6306
Penalty	549	512	250	430	307	381	208	322
Null	0.90	1.12	3.34	1.53	2.46	1.71	3.33	2.64
F.unuse	0.01	0.38	4.80	1.11	2.40	0.44	3.50	3.51
F.use	14.2	13.8	9.3	12.9	11.1	12.6	9.3	10.7
F.cost	4620	4594	4590	4570	4378	4233	4171	4627
F.aver	325	325	325	325	325	325	325	325
H.bonus	445	441	298	429	376	442	329	372
H.earn	7.1	6.9	4.7	6.5	5.5	6.3	6.6	9.1
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	1.9	3.7
H.use	7.1	6.9	4.7	6.5	5.5	6.3	4.7	5.4
H.cost	1421	1380	894	1302	1106	1300	1223	1679
H.aver	200.0	200.7	192.0	201.2	199.8	206.7	185.5	184.5

Table 7: Experiment with high mean, $\sigma = 20$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	956	841	-1029	474	-190	694	-827	-1000
Utility	7024	6820	4328	6346	5251	6320	4350	5370
Cost	6068	5978	5357	5871	5441	5625	5177	6371
Penalty	541	521	248	387	285	366	163	349
Null	0.89	1.09	3.69	1.66	2.79	1.73	3.77	2.66
F.unuse	0.04	0.38	5.53	1.60	3.28	0.86	4.10	3.56
F.use	14.2	13.8	8.6	12.7	10.4	12.5	8.4	10.7
F.cost	4637	4618	4597	4640	4454	4358	4080	4629
F.aver	325	325	325	325	325	325	325	325
H.bonus	452	429	269	397	323	414	289	380
H.earn	7.1	6.9	4.3	6.3	5.2	6.3	6.8	10.5
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	2.5	5.2
H.use	7.1	6.9	4.3	6.3	5.2	6.3	4.3	5.3
H.cost	1431	1360	761	1231	987	1268	1097	1742
H.aver	200.6	196.6	176.7	194.4	189.3	201.8	160.8	165.4

Table 8: Experiment with high mean, $\sigma = 40$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	941	829	-1190	308	-365	608	-971	-1017
Utility	6985	6801	4064	6053	5011	6225	4065	5202
Cost	6044	5972	5255	5745	5377	5617	5037	6220
Penalty	535	520	234	357	267	352	136	341
Null	0.93	1.12	3.96	1.97	3.02	1.82	4.06	2.82
F.unuse	0.03	0.39	6.05	2.01	3.82	1.08	4.63	3.76
F.use	14.1	13.8	8.1	12.1	10.0	12.4	7.9	10.3
F.cost	4604	4601	4594	4571	4475	4367	4062	4587
F.aver	325	325	325	325	325	325	325	325
H.bonus	454	440	258	385	303	400	267	369
H.earn	7.1	6.9	4.0	6.0	5.0	6.2	7.1	11.0
H.unuse	0.1	0.1	0.0	0.0	0.0	0.0	3.0	5.8
H.use	7.1	6.9	4.0	6.0	5.0	6.2	4.0	5.2
H.cost	1440	1371	661	1174	901	1249	975	1633
H.aver	202.1	197.9	163.7	194.8	181.3	201.6	138.0	148.9

Table 9: Experiment with high mean, $\sigma = 60$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	901	815	-1164	272	-441	387	-946	-871
Utility	6912	6774	4014	5973	4879	5972	3928	5272
Cost	6011	5958	5178	5700	5320	5585	4875	6143
Penalty	507	501	226	330	254	339	129	349
Null	1.06	1.18	4.02	2.09	3.16	2.05	4.20	2.75
F.unuse	0.11	0.42	6.07	2.17	4.02	1.71	4.68	3.57
F.use	13.9	13.6	7.9	11.8	9.7	11.9	7.6	10.5
F.cost	4546	4567	4556	4546	4453	4420	3992	4573
F.aver	325	325	325	325	325	325	325	325
H.bonus	481	459	266	396	292	367	256	373
H.earn	7.1	6.9	4.0	5.9	4.8	6.0	7.5	11.7
H.unuse	0.2	0.1	0.0	0.0	0.0	0.0	3.6	6.5
H.use	6.9	6.8	4.0	5.9	4.8	6.0	3.9	5.2
H.cost	1465	1391	623	1154	867	1165	883	1571
H.aver	205.4	201.0	156.8	195.4	179.2	194.4	118.1	133.8

Table 10: Experiment with high mean, $\sigma = 80$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	841	766	-1107	252	-430	263	-924	-868
Utility	6752	6644	3891	5779	4822	5760	3863	5061
Cost	5910	5878	4998	5527	5252	5496	4788	5929
Penalty	476	460	219	319	260	316	124	319
Null	1.29	1.39	4.18	2.33	3.22	2.29	4.27	2.99
F.unuse	0.21	0.45	5.93	2.21	4.04	2.01	4.74	3.53
F.use	13.4	13.2	7.6	11.3	9.6	11.4	7.5	10.0
F.cost	4431	4443	4410	4402	4421	4367	3963	4405
F.aver	325	325	325	325	325	325	325	325
H.bonus	516	496	291	434	301	363	262	367
H.earn	7.1	6.8	3.8	5.7	4.8	5.8	7.7	12.2
H.unuse	0.4	0.2	0.0	0.0	0.0	0.0	3.8	7.2
H.use	6.7	6.6	3.8	5.7	4.8	5.7	3.8	5.0
H.cost	1479	1435	588	1125	831	1129	825	1524
H.aver	208.1	209.5	154.0	198.7	173.8	196.2	107.7	125.3

Table 11: Experiment with high mean, $\sigma = 100$

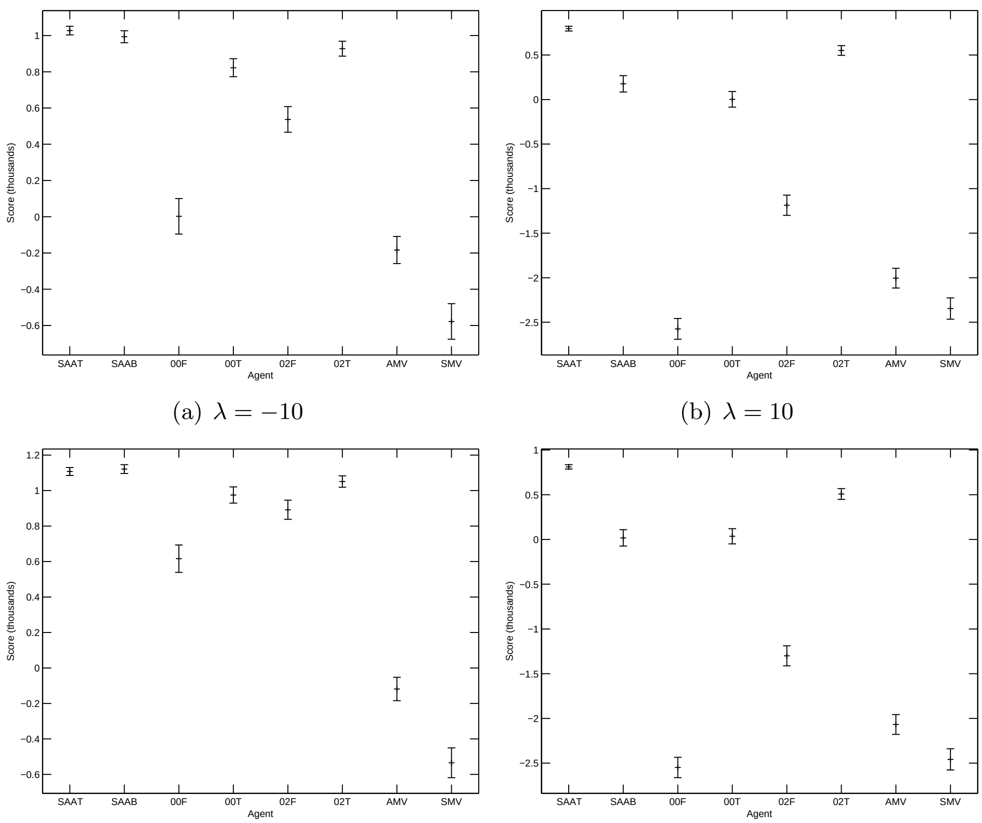
Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	720	733	-1150	152	-551	77	-996	-823
Utility	6489	6553	3686	5556	4604	5522	3688	4993
Cost	5768	5820	4836	5403	5156	5445	4684	5816
Penalty	435	432	199	275	246	301	111	293
Null	1.60	1.53	4.41	2.62	3.43	2.54	4.46	3.08
F.unuse	0.39	0.50	5.96	2.44	4.30	2.59	4.93	3.44
F.use	12.8	12.9	7.2	10.8	9.1	10.9	7.1	9.8
F.cost	4285	4364	4271	4291	4365	4392	3904	4313
F.aver	325	325	325	325	325	325	325	325
H.bonus	526	520	296	449	284	360	260	373
H.earn	7.0	6.8	3.6	5.4	4.6	5.5	7.8	12.5
H.unuse	0.6	0.3	0.0	0.0	0.0	0.0	4.1	7.6
H.use	6.4	6.5	3.6	5.4	4.6	5.5	3.7	4.9
H.cost	1483	1456	565	1112	791	1053	780	1504
H.aver	212.1	214.8	157.5	206.7	173.2	192.0	100.2	120.6

Table 12: Experiment with high mean, $\sigma = 120$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	711	710	-928	122	-546	-56	-884	-791
Utility	6372	6371	3899	5392	4471	5351	3660	4955
Cost	5660	5661	4827	5269	5018	5407	4544	5746
Penalty	402	401	197	272	237	320	125	294
Null	1.76	1.76	4.23	2.79	3.57	2.71	4.48	3.13
F.unuse	0.45	0.50	5.50	2.53	4.25	2.96	4.57	3.22
F.use	12.5	12.5	7.5	10.4	8.9	10.6	7.0	9.7
F.cost	4200	4218	4235	4209	4259	4402	3772	4215
F.aver	325	325	325	325	325	325	325	325
H.bonus	537	533	331	454	284	382	269	378
H.earn	6.9	6.7	3.8	5.2	4.4	5.3	8.1	13.0
H.unuse	0.7	0.4	0.0	0.0	0.0	0.0	4.5	8.1
H.use	6.2	6.2	3.8	5.2	4.4	5.3	3.6	4.9
H.cost	1459	1442	592	1060	759	1005	772	1531
H.aver	210.5	216.5	157.2	203.5	171.5	189.6	95.5	118.3

Table 13: Experiment with high mean, $\sigma = 140$

B. Experiment with Noisy Prediction



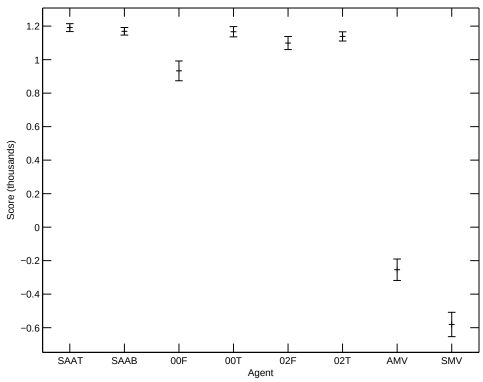
(a) $\lambda = -10$

(b) $\lambda = 10$

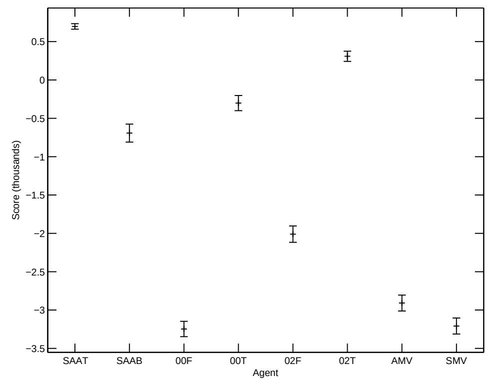
(c) $\lambda = -20$

(d) $\lambda = 20$

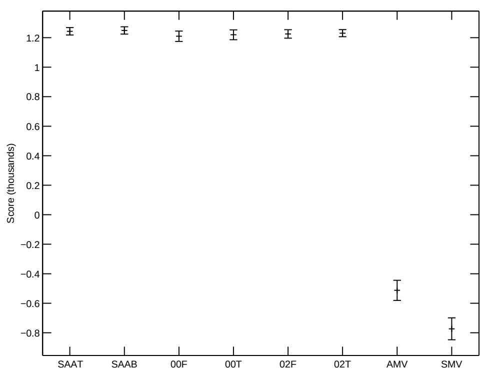
Figure 10: Confidence intervals of score mean, experiment with noise, $\sigma = 20$, I



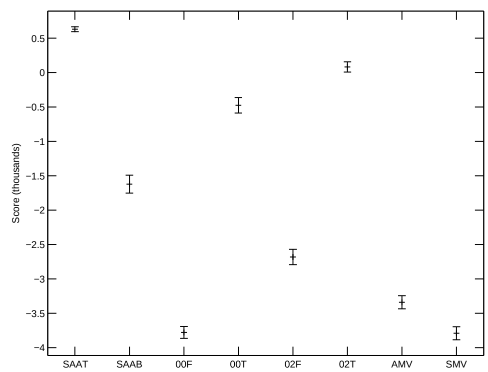
(a) $\lambda = -30$



(b) $\lambda = 30$



(c) $\lambda = -40$



(d) $\lambda = 40$

Figure 11: Confidence intervals of score mean, experiment with noise, $\sigma = 20$, II

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	1243	1249	1209	1219	1225	1230	-512	-773
Utility	6997	7019	6976	6914	6682	6631	6747	7252
Cost	5754	5770	5767	5695	5457	5401	7259	8025
Penalty	537	542	530	527	443	449	419	533
Null	0.91	0.89	0.95	0.99	1.31	1.39	1.46	0.88
F.unuse	0.00	0.00	0.15	0.07	0.07	0.00	0.12	0.05
F.use	14.2	14.2	14.1	14.0	13.4	13.2	13.1	14.2
F.cost	4607	4619	4635	4580	4368	4298	4289	4644
F.aver	325	325	325	325	325	325	325	325
H.bonus	448	456	453	433	440	469	626	666
H.earn	7.2	7.2	7.1	7.0	6.7	6.6	19.2	21.6
H.unuse	0.1	0.1	0.0	0.0	0.0	0.0	12.5	14.5
H.use	7.1	7.1	7.1	7.0	6.7	6.6	6.7	7.1
H.cost	1147	1151	1132	1115	1089	1103	2970	3382
H.aver	159.9	160.0	160.6	159.1	162.9	166.9	154.8	156.7

Table 14: Experiment with noise, $\sigma = 20$, $\lambda = -40$

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	1191	1169	932	1166	1098	1138	-254	-580
Utility	7075	6925	6647	7009	6699	6588	6612	7147
Cost	5884	5756	5714	5843	5600	5449	6866	7728
Penalty	541	527	506	514	460	436	398	526
Null	0.83	1.00	1.27	0.93	1.29	1.43	1.57	0.96
F.unuse	0.00	0.00	0.65	0.12	0.24	0.06	0.24	0.14
F.use	14.3	14.0	13.5	14.1	13.4	13.1	12.9	14.1
F.cost	4659	4553	4588	4633	4440	4289	4258	4625
F.aver	325	325	325	325	325	325	325	325
H.bonus	449	449	422	456	448	456	578	629
H.earn	7.3	7.1	6.7	7.1	6.7	6.6	16.1	18.9
H.unuse	0.1	0.0	0.0	0.0	0.0	0.0	9.5	11.8
H.use	7.2	7.0	6.7	7.1	6.7	6.6	6.5	7.0
H.cost	1225	1203	1126	1210	1161	1161	2608	3102
H.aver	168.9	170.8	167.4	171.2	173.0	176.7	162.3	164.4

Table 15: Experiment with noise, $\sigma = 20$, $\lambda = -30$

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	1107	1121	616	974	891	1051	-118	-534
Utility	7048	7056	6369	6868	6462	6541	6346	6822
Cost	5941	5935	5752	5893	5570	5490	6464	7356
Penalty	548	532	443	519	399	417	369	497
Null	0.85	0.87	1.59	1.06	1.57	1.50	1.80	1.25
F.unuse	0.00	0.02	1.39	0.40	0.67	0.14	0.74	0.60
F.use	14.3	14.3	12.8	13.9	12.9	13.0	12.4	13.5
F.cost	4648	4641	4622	4642	4394	4271	4272	4584
F.aver	325	325	325	325	325	325	325	325
H.bonus	447	457	399	443	435	459	513	566
H.earn	7.2	7.2	6.4	6.9	6.4	6.5	12.9	16.1
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	6.6	9.4
H.use	7.2	7.1	6.4	6.9	6.4	6.5	6.3	6.8
H.cost	1293	1294	1131	1251	1176	1219	2192	2773
H.aver	179.7	180.8	176.4	180.2	183.1	187.7	170.4	171.7

Table 16: Experiment with noise, $\sigma = 20$, $\lambda = -20$

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	1027	993	2	822	536	927	-183	-577
Utility	7018	6933	5619	6744	6016	6490	5798	6295
Cost	5991	5940	5617	5922	5479	5562	5982	6873
Penalty	553	524	356	476	362	418	301	420
Null	0.86	0.98	2.39	1.22	2.01	1.56	2.33	1.77
F.unuse	0.00	0.12	2.85	0.67	1.39	0.29	1.82	1.73
F.use	14.3	14.0	11.2	13.6	12.0	12.9	11.3	12.5
F.cost	4638	4599	4576	4628	4346	4281	4277	4614
F.aver	325	325	325	325	325	325	325	325
H.bonus	436	440	362	437	388	466	429	482
H.earn	7.2	7.0	5.6	6.8	6.0	6.4	9.6	12.6
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	3.9	6.4
H.use	7.1	7.0	5.6	6.8	6.0	6.4	5.7	6.2
H.cost	1353	1340	1041	1294	1134	1281	1705	2260
H.aver	189.0	190.6	185.5	190.8	189.3	199.0	177.7	179.2

Table 17: Experiment with noise, $\sigma = 20$, $\lambda = -10$

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	796	176	-2574	2	-1186	550	-2004	-2346
Utility	6932	6144	2613	5916	4010	6146	2803	3087
Cost	6136	5967	5188	5914	5197	5595	4808	5433
Penalty	539	430	67	311	156	327	58	118
Null	0.97	1.83	5.49	2.16	4.09	1.96	5.32	4.99
F.unuse	0.08	1.88	9.34	2.56	5.61	0.92	7.55	8.38
F.use	14.1	12.3	5.0	11.7	7.8	12.1	5.4	6.0
F.cost	4594	4622	4668	4625	4365	4229	4197	4680
F.aver	325	325	325	325	325	325	325	325
H.bonus	442	406	169	392	257	429	180	196
H.earn	7.0	6.2	2.5	5.8	3.9	6.0	3.0	3.8
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.8
H.use	7.0	6.2	2.5	5.8	3.9	6.0	2.7	3.0
H.cost	1541	1345	519	1288	831	1366	611	753
H.aver	219.3	218.2	207.0	220.8	212.7	226.1	203.3	198.6

Table 18: Experiment with noise, $\sigma = 20$, $\lambda = 10$

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	811	17	-2548	35	-1299	507	-2067	-2457
Utility	6980	5913	2606	5965	3903	6183	2728	2898
Cost	6169	5895	5154	5929	5203	5675	4796	5356
Penalty	526	439	74	322	153	327	52	98
Null	0.93	2.01	5.48	2.10	4.20	1.91	5.39	5.18
F.unuse	0.04	2.20	9.23	2.43	5.90	1.05	7.75	8.60
F.use	14.1	12.0	5.0	11.8	7.6	12.2	5.2	5.6
F.cost	4611	4609	4638	4623	4390	4302	4215	4625
F.aver	325	325	325	325	325	325	325	325
H.bonus	435	363	161	392	253	419	174	180
H.earn	7.1	6.0	2.5	5.9	3.8	6.1	2.9	3.7
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.9
H.use	7.1	6.0	2.5	5.9	3.8	6.1	2.6	2.8
H.cost	1558	1287	515	1307	813	1373	581	730
H.aver	220.3	214.9	204.8	221.7	213.8	225.5	204.1	198.5

Table 19: Experiment with noise, $\sigma = 20$, $\lambda = 20$

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	698	-692	-3247	-301	-2010	309	-2908	-3208
Utility	6941	5026	1680	5633	2988	5952	1703	1811
Cost	6242	5719	4927	5934	4999	5643	4612	5020
Penalty	532	343	24	256	85	281	23	41
Null	0.97	2.94	6.40	2.46	5.12	2.15	6.38	6.26
F.unuse	0.15	4.02	10.92	3.27	7.66	1.46	9.86	10.69
F.use	14.1	10.1	3.2	11.1	5.8	11.7	3.2	3.5
F.cost	4618	4596	4588	4662	4362	4277	4256	4605
F.aver	325	325	325	325	325	325	325	325
H.bonus	446	312	107	352	193	386	109	113
H.earn	7.0	5.1	1.6	5.5	2.9	5.8	1.7	2.0
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3
H.use	7.0	5.1	1.6	5.5	2.9	5.8	1.6	1.7
H.cost	1625	1124	339	1272	637	1366	356	415
H.aver	231.3	222.3	212.7	229.8	221.2	233.7	210.8	207.6

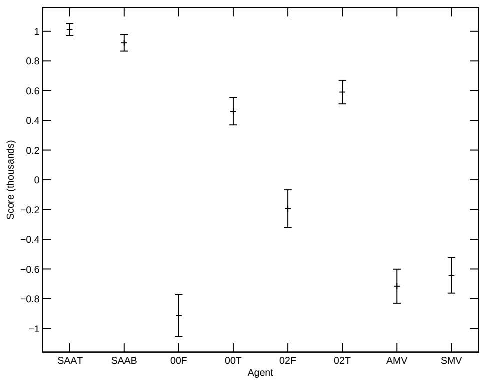
Table 20: Experiment with noise, $\sigma = 20$, $\lambda = 30$

Agent	SAAT	SAAB	00F	00T	02F	02T	AMV	SMV
Score	630	-1622	-3779	-476	-2681	82	-3340	-3791
Utility	6966	3884	1068	5420	2177	5711	1074	1109
Cost	6336	5506	4847	5896	4859	5628	4414	4900
Penalty	530	233	6	230	46	266	8	17
Null	0.95	4.13	6.99	2.68	5.92	2.42	6.99	6.95
F.unuse	0.17	6.47	12.22	3.61	9.32	1.90	10.86	12.21
F.use	14.1	7.7	2.0	10.6	4.2	11.2	2.0	2.1
F.cost	4638	4620	4627	4628	4385	4248	4190	4653
F.aver	325	325	325	325	325	325	325	325
H.bonus	448	244	67	335	140	394	69	73
H.earn	7.1	3.9	1.0	5.3	2.1	5.6	1.0	1.1
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
H.use	7.1	3.9	1.0	5.3	2.1	5.6	1.0	1.1
H.cost	1697	886	220	1268	474	1380	224	247
H.aver	240.8	228.8	219.1	238.6	227.6	247.2	219.0	217.5

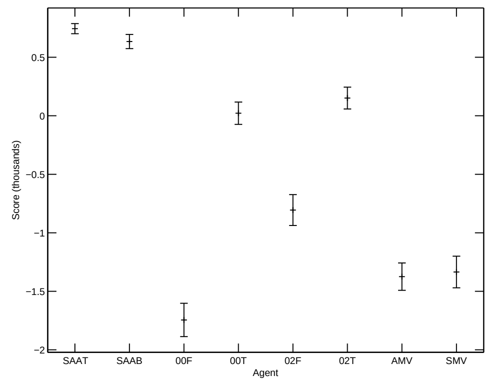
Table 21: Experiment with noise, $\sigma = 20$, $\lambda = 40$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	1186	1176	-115	857	337	812	-236	-332
Utility	6946	6956	5115	6426	5645	6265	4977	6263
Cost	5759	5780	5231	5568	5308	5453	5214	6595
Penalty	507	524	332	409	340	395	182	443
Null	1.06	1.01	2.90	1.60	2.35	1.74	3.21	1.78
F.unuse	0.02	0.08	3.71	1.16	2.39	1.04	2.66	1.52
F.use	13.9	14.0	10.2	12.8	11.3	12.5	9.6	12.4
F.cost	4514	4568	4518	4534	4447	4408	3981	4534
F.aver	325	325	325	325	325	325	325	325
H.bonus	518	491	352	437	340	399	367	491
H.earn	7.3	7.3	5.1	6.4	5.6	6.3	11.8	17.2
H.unuse	0.4	0.3	0.0	0.0	0.0	0.0	6.7	11.0
H.use	6.9	7.0	5.1	6.4	5.6	6.3	5.1	6.2
H.cost	1246	1212	713	1034	860	1044	1234	2061
H.aver	170.5	166.3	140.0	161.7	152.4	166.3	104.3	119.8

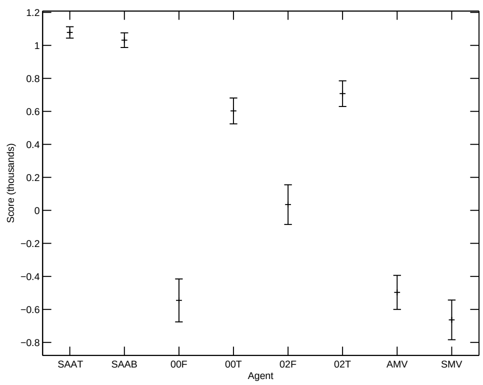
Table 22: Experiment with noise, $\sigma = 80$, $\lambda = -40$



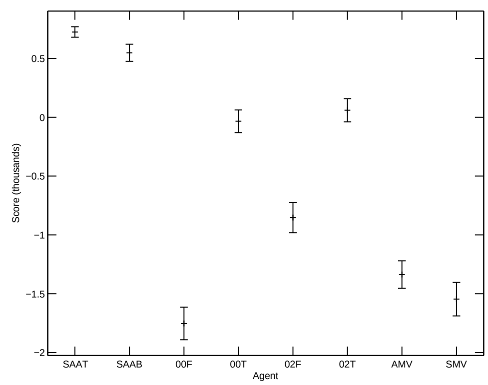
(a) $\lambda = -10$



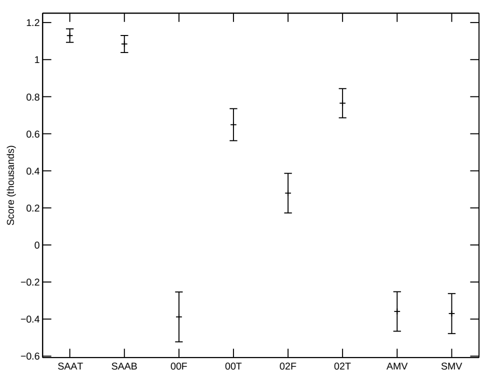
(b) $\lambda = 10$



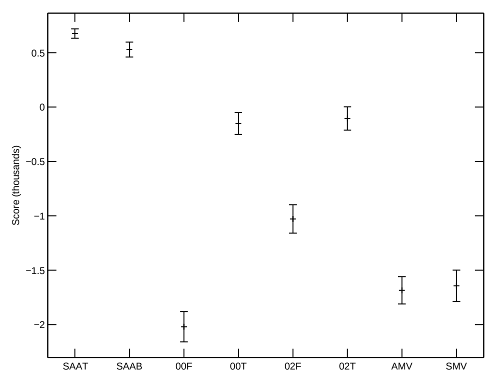
(c) $\lambda = -20$



(d) $\lambda = 20$

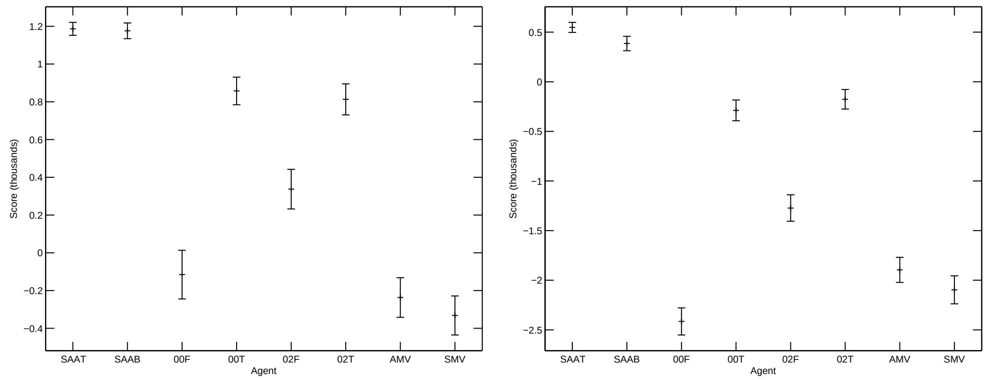


(e) $\lambda = -30$



(f) $\lambda = 30$

Figure 12: Confidence Intervals of score mean, experiment with noise, $\sigma = 80$, I



(a) $\lambda = -40$ (b) $\lambda = 40$

Figure 13: Confidence Intervals of score mean, experiment with noise, $\sigma = 80$, II

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	1129	1084	-388	648	279	764	-359	-370
Utility	6942	6904	4816	6338	5658	6236	4840	6176
Cost	5813	5820	5204	5689	5378	5471	5199	6546
Penalty	512	514	308	397	345	373	168	427
Null	1.04	1.06	3.22	1.69	2.33	1.77	3.33	1.88
F.unuse	0.05	0.18	4.28	1.55	2.46	1.08	3.09	1.78
F.use	13.9	13.9	9.6	12.6	11.3	12.5	9.3	12.2
F.cost	4538	4572	4501	4605	4482	4399	4039	4558
F.aver	325	325	325	325	325	325	325	325
H.bonus	497	477	343	429	338	384	340	482
H.earn	7.3	7.2	4.8	6.3	5.7	6.3	10.7	16.0
H.unuse	0.3	0.2	0.0	0.0	0.0	0.0	5.8	9.9
H.use	7.0	6.9	4.8	6.3	5.7	6.2	5.0	6.1
H.cost	1274	1247	704	1085	896	1072	1160	1988
H.aver	174.9	174.1	147.3	172.0	158.2	171.5	108.3	124.0

Table 23: Experiment with noise, $\sigma = 80$, $\lambda = -30$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	1078	1031	-545	602	35	707	-497	-663
Utility	6954	6833	4649	6232	5376	6256	4616	5704
Cost	5876	5801	5195	5629	5341	5549	5113	6368
Penalty	510	505	287	363	307	374	159	386
Null	1.04	1.13	3.39	1.83	2.64	1.74	3.55	2.34
F.unuse	0.04	0.20	4.63	1.58	3.02	1.08	3.41	2.48
F.use	13.9	13.7	9.2	12.3	10.7	12.5	8.9	11.3
F.cost	4540	4531	4502	4527	4467	4417	3999	4482
F.aver	325	325	325	325	325	325	325	325
H.bonus	501	468	328	424	322	375	328	435
H.earn	7.3	7.0	4.6	6.2	5.4	6.3	10.0	14.9
H.unuse	0.3	0.2	0.0	0.0	0.0	0.0	5.3	9.2
H.use	7.0	6.9	4.6	6.2	5.4	6.3	4.7	5.7
H.cost	1336	1271	693	1103	875	1131	1114	1885
H.aver	184.0	180.5	150.5	178.7	163.2	179.3	111.9	126.7

Table 24: Experiment with noise, $\sigma = 80$, $\lambda = -20$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	1010	921	-913	460	-194	590	-715	-642
Utility	6916	6749	4320	6114	5134	6153	4308	5635
Cost	5905	5827	5233	5653	5328	5563	5023	6277
Penalty	498	487	257	352	288	361	142	364
Null	1.09	1.22	3.72	1.95	2.90	1.87	3.86	2.42
F.unuse	0.08	0.34	5.53	1.87	3.57	1.37	4.03	2.72
F.use	13.8	13.6	8.6	12.1	10.2	12.3	8.3	11.2
F.cost	4520	4519	4580	4538	4477	4427	4003	4508
F.aver	325	325	325	325	325	325	325	325
H.bonus	503	455	296	419	321	389	306	422
H.earn	7.1	6.9	4.3	6.0	5.1	6.1	8.8	13.4
H.unuse	0.2	0.1	0.0	0.0	0.0	0.0	4.4	7.8
H.use	6.9	6.8	4.3	6.0	5.1	6.1	4.3	5.6
H.cost	1385	1309	653	1115	851	1137	1021	1769
H.aver	193.9	189.5	152.7	184.4	166.8	185.0	116.4	131.8

Table 25: Experiment with noise, $\sigma = 80$, $\lambda = -10$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	744	634	-1744	21	-805	151	-1374	-1335
Utility	6787	6599	3329	5753	4478	5734	3369	4520
Cost	6043	5964	5074	5731	5284	5583	4744	5855
Penalty	491	480	184	303	223	288	105	290
Null	1.21	1.36	4.71	2.34	3.57	2.34	4.75	3.50
F.unuse	0.18	0.63	7.36	2.68	4.80	2.11	5.81	4.80
F.use	13.6	13.3	6.6	11.3	8.9	11.3	6.5	9.0
F.cost	4471	4523	4528	4550	4440	4367	4003	4485
F.aver	325	325	325	325	325	325	325	325
H.bonus	493	437	229	395	272	360	223	313
H.earn	6.9	6.7	3.3	5.7	4.4	5.7	5.7	9.5
H.unuse	0.1	0.0	0.0	0.0	0.0	0.0	2.4	5.0
H.use	6.8	6.6	3.3	5.7	4.4	5.7	3.3	4.5
H.cost	1572	1441	546	1181	844	1216	741	1370
H.aver	226.9	215.6	166.3	208.7	190.6	213.7	130.1	143.9

Table 26: Experiment with noise, $\sigma = 80$, $\lambda = 10$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	725	548	-1753	-33	-853	60	-1337	-1546
Utility	6749	6579	3255	5697	4433	5579	3320	4249
Cost	6024	6031	5008	5731	5287	5519	4657	5796
Penalty	505	488	176	290	221	279	97	271
Null	1.21	1.39	4.78	2.40	3.61	2.48	4.80	3.77
F.unuse	0.22	0.83	7.39	2.76	4.95	2.39	5.72	5.39
F.use	13.6	13.2	6.4	11.2	8.8	11.0	6.4	8.5
F.cost	4481	4567	4493	4538	4466	4365	3941	4502
F.aver	325	325	325	325	325	325	325	325
H.bonus	469	458	214	388	261	338	214	290
H.earn	6.9	6.7	3.2	5.6	4.4	5.5	5.8	9.4
H.unuse	0.1	0.1	0.0	0.0	0.0	0.0	2.5	5.1
H.use	6.8	6.6	3.2	5.6	4.4	5.5	3.3	4.2
H.cost	1542	1464	515	1192	821	1153	716	1294
H.aver	223.0	218.5	160.3	213.0	187.0	208.4	124.5	138.4

Table 27: Experiment with noise, $\sigma = 80$, $\lambda = 20$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	675	528	-2019	-150	-1028	-105	-1684	-1643
Utility	6825	6624	2993	5576	4223	5544	2975	4027
Cost	6149	6096	5013	5726	5251	5649	4660	5671
Penalty	505	474	166	276	203	286	87	245
Null	1.14	1.35	5.04	2.53	3.83	2.52	5.13	4.00
F.unuse	0.22	0.78	8.01	2.99	5.29	2.69	6.67	5.86
F.use	13.7	13.3	5.9	10.9	8.3	11.0	5.7	8.0
F.cost	4532	4576	4527	4528	4428	4437	4033	4505
F.aver	325	325	325	325	325	325	325	325
H.bonus	469	450	201	381	261	348	191	273
H.earn	7.0	6.7	3.0	5.5	4.2	5.5	4.8	8.1
H.unuse	0.1	0.1	0.0	0.0	0.0	0.0	1.9	4.1
H.use	6.9	6.7	3.0	5.5	4.2	5.5	2.9	4.0
H.cost	1617	1520	486	1198	824	1212	627	1166
H.aver	232.3	226.3	164.5	219.1	197.8	220.2	131.5	144.2

Table 28: Experiment with noise, $\sigma = 80$, $\lambda = 30$

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	548	386	-2415	-288	-1272	-176	-1895	-2096
Utility	6729	6402	2524	5417	3908	5455	2679	3401
Cost	6181	6015	4939	5705	5180	5631	4575	5498
Penalty	494	447	130	253	180	249	75	211
Null	1.25	1.60	5.52	2.70	4.14	2.62	5.42	4.63
F.unuse	0.38	1.03	8.92	3.22	5.82	2.77	7.21	7.06
F.use	13.5	12.8	5.0	10.6	7.7	10.8	5.2	6.7
F.cost	4508	4498	4510	4490	4401	4395	4020	4488
F.aver	325	325	325	325	325	325	325	325
H.bonus	477	445	178	372	229	327	177	239
H.earn	6.8	6.4	2.5	5.3	3.9	5.4	3.9	6.6
H.unuse	0.1	0.0	0.0	0.0	0.0	0.0	1.3	3.3
H.use	6.7	6.4	2.5	5.3	3.9	5.4	2.6	3.4
H.cost	1673	1517	429	1215	780	1236	555	1010
H.aver	244.4	235.8	173.7	229.4	202.1	228.5	142.0	152.3

Table 29: Experiment with noise, $\sigma = 80$, $\lambda = 40$

C. Experiment with Equilibrium Prediction

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	944	863	-338	562	707	820	-1353	-1290
Utility	6536	6391	4756	6002	4617	5104	4528	4818
Cost	5592	5528	5094	5440	3909	4284	5881	6108
Penalty	426	423	261	328	133	178	198	261
Null	1.45	1.59	3.28	2.06	3.56	3.06	3.64	3.32
F.unuse	0.00	0.22	3.61	1.21	0.32	0.18	3.78	3.73
F.use	13.1	12.8	9.4	11.9	8.9	9.9	8.7	9.4
F.cost	4255	4235	4239	4253	2991	3272	4062	4256
F.aver	325	325	325	325	325	325	325	325
H.bonus	417	408	301	391	310	339	367	396
H.earn	6.6	6.4	4.7	5.9	4.4	4.9	11.2	11.1
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	6.7	6.4
H.use	6.5	6.4	4.7	5.9	4.4	4.9	4.5	4.7
H.cost	1336	1294	856	1186	918	1012	1820	1852
H.aver	203.6	201.8	181.5	199.8	207.0	204.6	162.6	167.2

Table 30: Experiment with equilibrium, decision-theoretic setting, B(32, 0.5) players

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	2516	2019	904	2520	1989	2492	1447	1233
Utility	8579	8010	6717	8577	7703	8387	7276	7144
Cost	6063	5991	5813	6056	5713	5895	5829	5911
Penalty	46	56	53	61	363	230	82	57
Null	0.00	0.51	1.72	0.00	0.54	0.00	1.13	1.30
F.unuse	0.00	1.02	3.43	0.00	1.08	0.00	2.16	2.60
F.use	16.0	15.0	12.6	16.0	14.9	16.0	13.7	13.4
F.cost	5200	5200	5200	5200	5200	5200	5165	5201
F.aver	325	325	325	325	325	325	325	325
H.bonus	627	576	488	639	607	618	490	499
H.earn	16.0	15.4	12.8	15.1	11.2	13.8	17.2	15.5
H.unuse	0.5	1.2	1.8	0.0	0.3	0.0	5.1	3.6
H.use	15.5	14.2	11.0	15.1	10.9	13.8	12.1	12.0
H.cost	863	791	613	856	513	695	664	710
H.aver	53.8	51.4	47.8	56.6	45.7	50.6	38.7	45.7

Table 31: Experiment with equilibrium, game-theoretic setting, 8 players

Agent	SAAT	SAAB	TMU	TMU*	BE	BE*	AMU	SMU
Score	1041	1035	418	867	899	943	-721	-670
Utility	6520	6519	5653	6304	4765	5089	5664	6012
Cost	5479	5484	5235	5437	3865	4145	6385	6683
Penalty	437	427	316	382	137	181	267	341
Null	1.46	1.47	2.39	1.71	3.41	3.07	2.52	2.12
F.unuse	0.00	0.05	1.83	0.58	0.07	0.02	1.56	1.34
F.use	13.1	13.1	11.2	12.6	9.2	9.9	11.0	11.8
F.cost	4250	4261	4244	4275	3002	3215	4070	4257
F.aver	325	325	325	325	325	325	325	325
H.bonus	420	420	357	402	317	336	447	477
H.earn	6.6	6.6	5.6	6.3	4.6	4.9	14.0	14.3
H.unuse	0.0	0.0	0.0	0.0	0.0	0.0	8.4	8.4
H.use	6.5	6.5	5.6	6.3	4.6	4.9	5.7	5.9
H.cost	1229	1223	991	1162	863	930	2315	2426
H.aver	186.7	186.5	176.6	184.9	188.3	188.4	164.9	169.8

Table 32: Experiment with equilibrium, game-theoretic setting, B(32, 0.5) players